Example: By using the bisection method find an approximation root of the equation $x^3 - x - 1.225 = 0$ that lies between $a = 1$ and $b = 2$. Cary out computation upto the third stage (or $\epsilon = 0.001$)

Solution:

Let $f(x) = x^3 - x - 1.225$

$f(1) = 1^3 - 1 - 1.225 = -1.225$ (−ve)

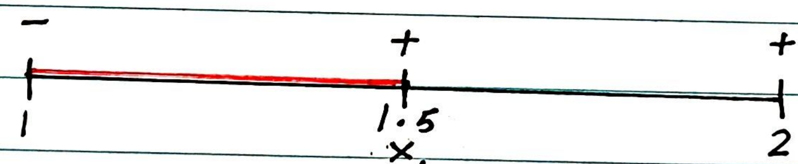
$f(2) = 2^3 - 2 - 1.225 = 4.775$ (+ve)

$\therefore$ a root lies between 1 and 2.

Thus the first approximation to the root is

$x_1 = \frac{1+2}{2} = 1.5$

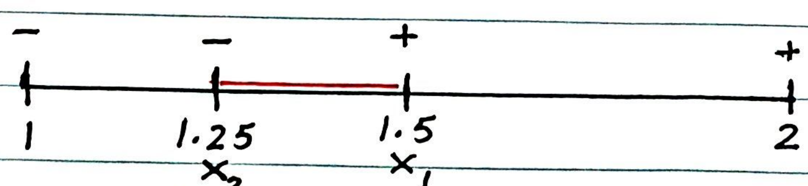
and $f(x_1) = (1.5)^3 - 1.5 - 1.225 = 3.375 - 1.5 - 1.225$
$= 0.65$ (+ve)



Hence the root lies between 1 and $x_1 = 1.5$.

Thus the second approximation to the root is $x_2 = \frac{1+1.5}{2} = 1.25$

and $f(x_2) = (1.25)^3 - 1.25 - 1.225$
$= 1.953125 - 1.25 - 1.225 = -0.521875$ (−ve)



Hence the root lies between $x_2 = 1.25$ and $x_1 = 1.5$.

Thus the third approximation to the root is

$$x_3 = \frac{1.25+1.5}{2} = 1.375$$

and $f(x_3) = (1.375)^3 - 1.375 - 1.225$

$= 2.599609375 - 1.375 - 1.225 = -0.00039062501$

$\therefore$ The root $\bar{x}$ is approximately equal 1.375, since $|f(x_3)| = 0.00039062501 < 0.001$

Example : By using the bisection method find an approximation root of the equation $e^{-x} = x$ and $\epsilon = 6\%$.

Solution : $e^{-x} = x \Rightarrow e^{-x} - x = 0$

Let $f(x) = e^{-x} - x$

$f(0) = e^{-0} - 0 = 1 \qquad (+ve)$

$f(1) = e^{-1} - 1 = 0.367879441 - 1 = -0.632120558$

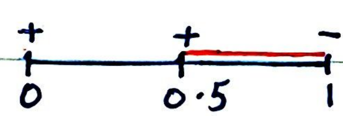
$(-ve)$

$\therefore$ $a = 0$ and $b = 1$ and there is a root in the interval $[0, 1]$.

$$\therefore x_1 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$f(x_1) = e^{-0.5} - 0.5 = 0.606530659 - 0.5$

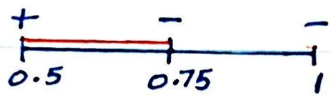
$= 0.106530659 \qquad (+ve)$



$$\therefore x_2 = \frac{0.5+1}{2} = 0.75$$

$f(x_2) = e^{-0.75} - 0.75 = 0.472366552 - 0.75$

$= -0.277633447 \qquad (-ve)$

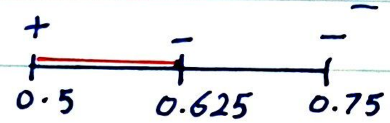


$$e\% = \left|\frac{0.75 - 0.5}{0.75}\right| * 100 \simeq 33.33\,\%$$

$$\therefore\ x_3 = \frac{0.5 + 0.75}{2} = 0.625$$

$$f(x_3) = e^{-0.625} - 0.625 = 0.535261428 - 0.625$$

$$= -0.089738571 \quad (-ve)$$

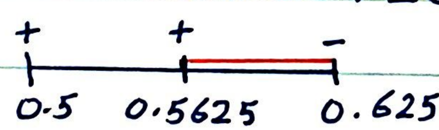


$$e\% = \left|\frac{0.625 - 0.75}{0.625}\right| * 100 = 20\%$$

$$\therefore\ x_4 = \frac{0.5 + 0.625}{2} = 0.5625$$

$$f(x_4) = e^{-0.5625} - 0.5625 = 0.569782824 - 0.5625$$

$$= 0.007282824732 \quad (+ve)$$



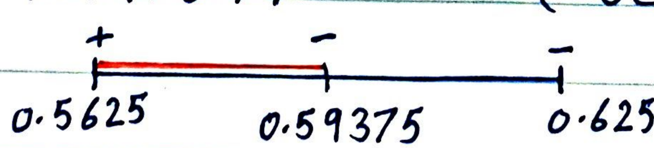
$$e\% = \left|\frac{0.5625 - 0.625}{0.5625}\right| * 100 \simeq 11.11\,\%$$

$$\therefore\ x_5 = \frac{0.5625 + 0.625}{2} = 0.59375$$

$$f(x_5) = e^{-0.59375} - 0.59375$$

$$= 0.55225245 - 0.59375$$

$$= -0.041497549 \qquad (-ve)$$



$$e\% = \left| \frac{0.59375 - 0.5625}{0.59375} \right| * 100 \simeq 5.26\% < \epsilon$$

$\therefore$ The root $\bar{x}$ is approximately equal 0.59375 .

We can summarize the above results in the following table:

i	a	b	x_i	$e\%$	$f(a)$	$f(x_i)$	$f(a)*f(x_i)$
1	0	1	0.5	—	+	+	+
2	0.5	1	0.75	$\simeq 33.33\%$	+	−	−
3	0.5	0.75	0.625	20%	+	−	−
4	0.5	0.625	0.5625	$\simeq 11.11\%$	+	+	+
5	0.5625	0.625	0.59375	$\simeq 5.26\%$	+	−	−

$\therefore$ The root is $\bar{x} \simeq 0.59375$.

Exercises:

1) Find the approximate root of the following equation by using the bisection method:
$3x^2 = x + 5.25$ in the interval $[0.5, 3]$ and $\epsilon = 0.4$.