Higher order Derivatives :-

المشتقات ذات الرتب العليا

1. The second order partial derivative which are denoted by :-

$\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ or

f_{xx}, f_{yy}, f_{xy}, f_{yx}.

Remark :- IF $f(x,y)$ have continuous partial derivative Then $f_{xy} = f_{yx}$.

2. $\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)\right)$, $\frac{\partial^3 f}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)\right)$

$\frac{\partial^3 f}{\partial x^2 \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\right)$, $\frac{\partial^3 f}{\partial y^2 \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right)$

3. IF f have cont. partial derivative then $\frac{\partial^3 f}{\partial x^2 \partial y} = \frac{\partial^3 f}{\partial y \partial x^2}$

4. IF all the partial derivative that apper are cont., the notation

$\frac{\partial^{m+n} f}{\partial x^m \partial y^n}$ may be used to denoted the result of differentia

The Function $f(x,y)$, m-times w.r.t x and n-times w.r.t y.

5. IF $f(x,y,r,s,u,z)$ have cont. partial derivative, then

$f_{xy} = f_{yx}$, $f_{xz} = f_{zx}$, $f_{xu} = f_{ux}$, $f_{zu} = f_{uz}$ .... etc.

6.
$$\frac{\partial^4 f}{\partial x \partial x \partial y \partial y} = \frac{\partial^4 f}{\partial x \partial y \partial x \partial y} = \frac{\partial^4 f}{\partial x \partial y \partial y \partial x} = \frac{\partial^4 f}{\partial y \partial x \partial x \partial y} =$$

$$\frac{\partial^4 f}{\partial y \partial x \partial y \partial x} = \frac{\partial^4 f}{\partial y \partial y \partial x \partial x}$$

Ex, Given $Z = x^3 + 3x^2y - 2x^2y - y^4 + 3xy$

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial x \partial y}$, $\frac{\partial^2 z}{\partial y \partial x}$.

Sol 1. $\frac{\partial z}{\partial x} = 3x^2 + 6xy - 4xy + 3y \Rightarrow 3x^2 + 2xy + 3y$

2. $\frac{\partial z}{\partial y} = 3x^2 - 2x^2 - 4y^3 + 3x \Rightarrow x^2 + 3x - 4y^3$

3. $\frac{\partial^2 z}{\partial x^2} = 6x + 6y - 4y \Rightarrow 6x + 2y.$

4. $\frac{\partial^2 z}{\partial y^2} = -12y^2$

5. $\frac{\partial^2 z}{\partial x \partial y} = 2x + 3.$

6. $\frac{\partial^2 z}{\partial y \partial x} = 2x + 3$

Exc 1- Let $f(x,y) = x\cos y + ye^x$

1. Find all First partial derivative 2. Find all second partial derivative 3. Find $\frac{\partial^3 f}{\partial x^2 \partial y}$, $\frac{\partial^3 f}{\partial x \partial y^2}$

2. Given $U = e^x \cos y + e^x \sin Z$.

Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$, $\frac{\partial^2 u}{\partial x \partial z} = \frac{\partial^2 u}{\partial z \partial x}$.

Exc (42) Higher Order Derivatives

let $f(x,y) = x\cos y + y e^{x}$

1- Find all first partial derivative

2- Find all second partial derivative

3- Find $\dfrac{\partial^3 f}{\partial x^2 \partial y}$, $\dfrac{\partial^3 f}{\partial x \partial y^2}$

solution :- ①

$$\frac{\partial f}{\partial x} = \cos y + y e^{x}$$

$$\frac{\partial f}{\partial y} = -x \sin y + e^{x}$$

②

$$\frac{\partial^2 f}{\partial x^2} = y e^{x}$$

$$\frac{\partial^2 f}{\partial y^2} = -x \cos y$$

③ $$\frac{\partial^3 f}{\partial x^2 \partial y} = e^{x}$$

$$\frac{\partial^3 f}{\partial x \partial y^2} = -\cos y$$

Exc — $u = e^{x}\cos y + e^{x}\sin z$

verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$,

$$\frac{\partial^2 u}{\partial x \partial z} = \frac{\partial^2 u}{\partial z \partial x}$$

solution

$$\frac{\partial u}{\partial y} = -e^{x}\sin y$$

$$\frac{\partial^2 u}{\partial x \partial y} = -e^{x}\sin y \quad --(1)$$

$$\frac{\partial u}{\partial x} = e^{x}\cos y + e^{x}\sin z$$

$$\frac{\partial u}{\partial y \partial x} = -e^{x}\sin y \quad ---(2)$$

$\therefore$ From (1) & (2) we get $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Now

$$\frac{\partial u}{\partial z} = e^{x}\cos z$$

$$\frac{\partial u}{\partial x \partial z} = e^{x}\cos z \quad —(1)$$

$$\frac{\partial u}{\partial x} = e^{x}\cos y + e^{x}\sin z$$

$$\frac{\partial u}{\partial z \partial x} = e^{x}\cos z \quad --(2)$$

From (1) & (2) we get

$$\frac{\partial u}{\partial x \partial z} = \frac{\partial u}{\partial z \partial x}$$