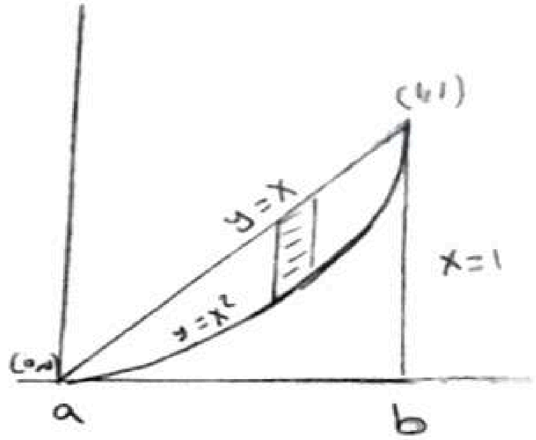
Ex Find the area of the region R bounded by $y = x$ and $y = x$ in the first quadrant. الربع الاول

Solution

$$A = \int_0^1 \int_{x^2}^{x} dy\, dx$$

$$= \int_0^1 (x - x^2)\, dx$$

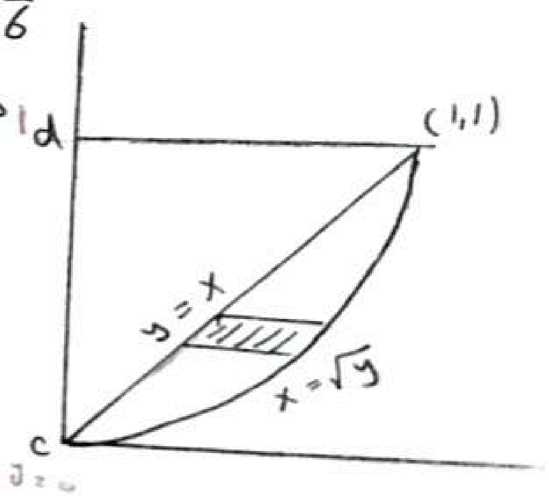
$$= \frac{x^2}{2} - \frac{x^3}{3} \Big]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$



we can find the area as follows

$$A = \int_0^1 \int_y^{\sqrt{y}} dx\, dy = \int_0^1 (\sqrt{y} - y)\, dy$$

$$= \frac{2}{3} y^{3/2} - \frac{y^2}{2} \Big]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$



Ex Find the area of the region enclosed by the parab $y = x^2$ and the Line $y = x + 2$.

Solution

$y = x^2$, $y = x + 2$ (المشتركة الرقمية)

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\Rightarrow x = 2 , x = -1$

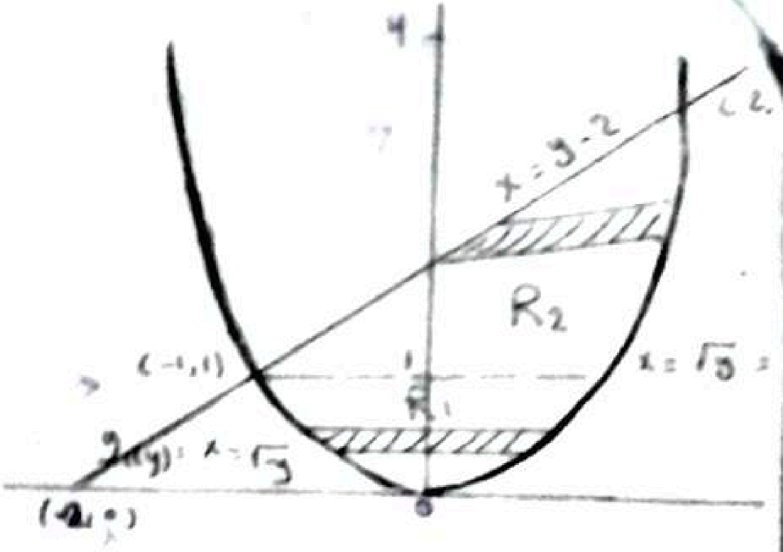
$\Rightarrow y = 4 , y = 1 \Big\} \Rightarrow (2,4), (-1,1)$

$\therefore (2,4), (-1,1)$ (هي نقاط تقاطع المنحنيات)

$$A = \iint_{R_1} dA + \iint_{R_2} dA$$

$$= \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy + \int_1^4 \int_{y-2}^{\sqrt{y}} dx\,dy$$

$$= \int_0^1 \Big[x\Big]_{-\sqrt{y}}^{\sqrt{y}} dy + \int_1^4 \Big[x\Big]_{y-2}^{\sqrt{y}} dy$$

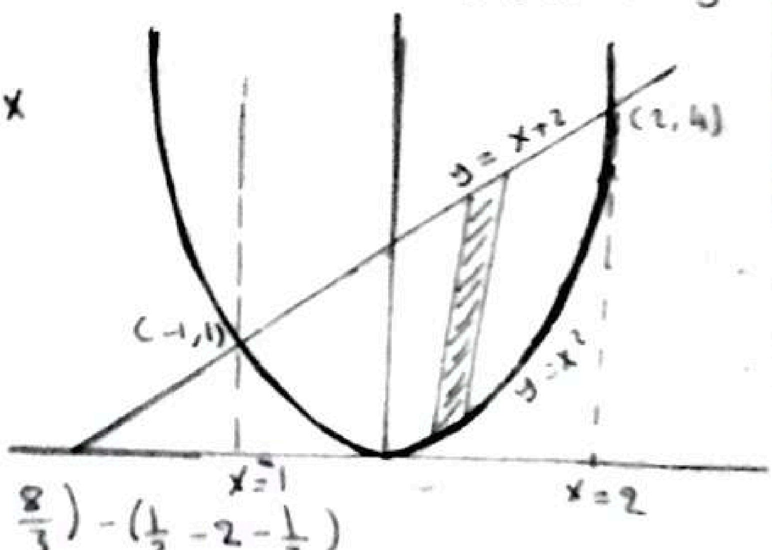


$$= \int_0^1 2\sqrt{y}\, dy + \int_1^4 (\sqrt{y} - y + 2)\, dy = \frac{2y^{3/2}}{\frac{3}{2}}\Big]_0^1 + \frac{2}{3}y^{3/2} - \frac{y^2}{2} + 2$$

$$= \frac{4}{3} + \left(\frac{16}{2} - 8 + 8\right) - \left(\frac{2}{3} - \frac{1}{2} + 2\right) = \frac{9}{2}$$

- We can find the area as follows (الشريحه العموديه)

$$\int_{-1}^{2}\int_{x^2}^{x+2} dy\,dx = \int_{-1}^{2} \Big[y\Big]_{x^2}^{x+2} dx$$



$$= \int_{-1}^{2} (x + 2 - x^2)\, dx$$

$$= \frac{x^2}{2} + 2x - \frac{x^3}{3}\Big]_{-1}^{2} = \left(4 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 - \frac{1}{3}\right)$$

$$= 8 - 3 - \frac{1}{2} = \frac{9}{2}$$