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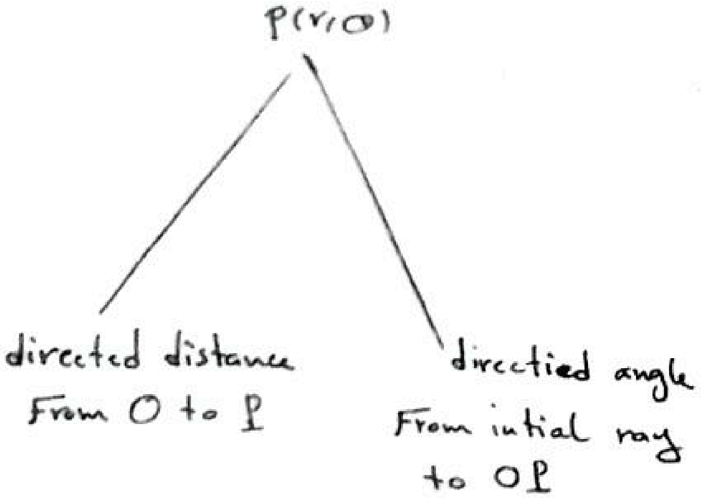
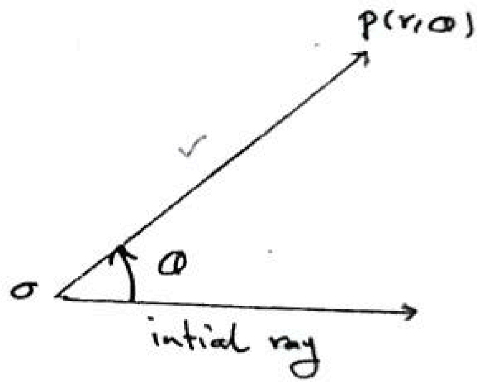
محاضرة ٣ مرحلة ثانية
رياضيات

Polar Coordinate :-

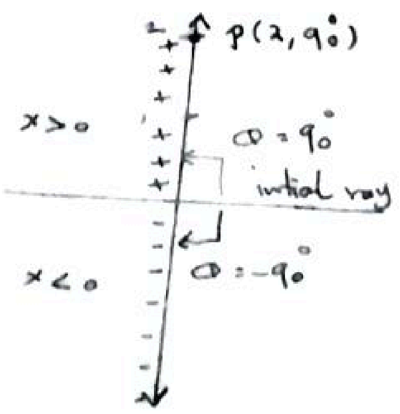
To define polar coordinate, we First fix an origin O and an intial ray From O, then each point P can be assigned polar coordinates (r, θ) in which the First number r, gives the directed distance From O to P and the Second number θ gives the directed angle From the intial ray to the segment OP.





Remark :- The angle θ is <u>positive</u> when measured Counter-clockwise (بعكس اتجاه عقارب الساعة) and <u>negetive</u> when measured clockwise (باتجاه عقارب الساعة)

Ex :- Plote (ارسم) the point $(2, 90^\circ)$



Remark (1) الملاحظات الرياضية

Any point P in Polar coordinate system with Coordinate (r, θ) has also infinite number of Polar coordinates which are

1) (a) $(r, \theta + n.360^\circ)$ for all integers n, when θ is measured in degrees

(b) $(-r, (\theta - 180) + n360^\circ)$ for all integers n, when θ is measured in degrees

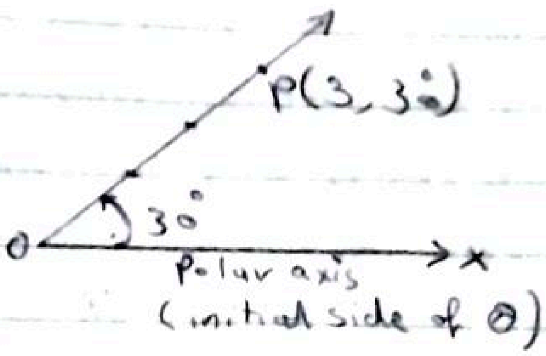
2) (a) $(r, \theta + 2n\pi)$ for all integers n when θ is measured in radiants

(b) $(-r, (\theta - \pi) + 2n\pi)$ for all integers n, when θ is measured in radiants.

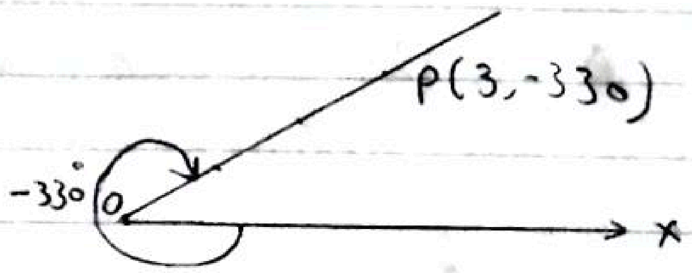
Example: Find all the polar coordinates (r, θ) of the point $P(3, 30^\circ)$ such that $-360 \leq \theta \leq 360$ and plot the point P for all these coordinates.

Solution: the point P has the coordinates $(3, 30^\circ)$, $(3, -330^\circ)$, $(-3, 210^\circ)$, $(-3, -150^\circ)$ i.e.
$P(3, 30^\circ) = P(3, -330^\circ) = P(-3, 210^\circ) = P(-3, -150^\circ)$.

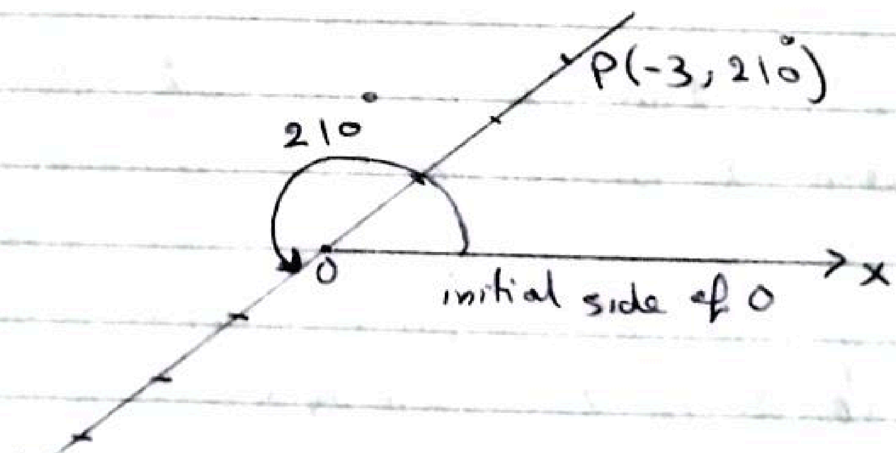
1- Plotting $P(3, 30^\circ)$



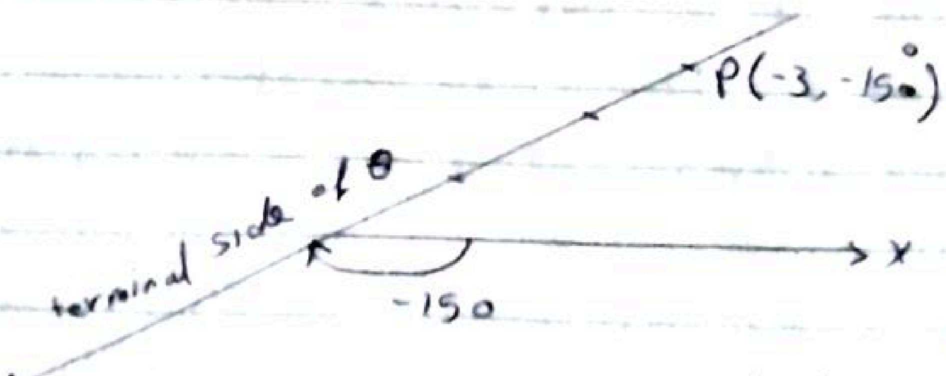
2. plotting $P(3, -330^\circ)$



3- plotting $(-3, 210^\circ)$



4) plotting $(-3, 150^\circ)$



Example :- Find all the polar coordinates of the point $p(2, 30^\circ)$

Solution :-

The polar coordinates are

$(2, 30^\circ + n.180^\circ)$, $n = 0, \mp 2, \mp 4, \mp 6$

i.e $(2, 30^\circ), (2, 390^\circ), (2, 750^\circ), (2, 1110^\circ), \dots;$

$(2, -330^\circ), (2, -690^\circ), (2, -1050^\circ), \dots,$

and $(-2, 30^\circ + n.180^\circ)$, $n = \mp 1, \mp 3, \mp 5, \dots$

i.e $(-2, 210^\circ), (-2, 570^\circ), (-2, 930^\circ), \dots,$

$(-2, -150^\circ), (-2, -510^\circ), (-2, -870^\circ).$

Example :- Find all the polar coordinates of the point $p(2, \frac{\pi}{6})$

sol

the polar coordinates are (1) when $\boxed{r = 2}$

$(2, \frac{\pi}{6} + n\pi)$, $n = 0, \mp 2, \mp 4, \mp 6, \dots$ i.e

$(2, \frac{\pi}{6}), (2, \frac{13\pi}{6}), (2, \frac{25\pi}{6}), \dots;$

$(2, \frac{-11\pi}{6}), (2, -\frac{23\pi}{6}), (2, -\frac{35\pi}{6}) \dots,$

and (2) when $\boxed{r = -2}$

$(-2, \frac{\pi}{6} + n\pi)$, $n = \mp 1, \mp 3, \mp 5, \dots$

i.e

$(-2, \frac{7\pi}{6}), (-2, \frac{19\pi}{6}), (-2, \frac{31\pi}{6}), \dots;$

$(-2, -\frac{5\pi}{6}), (-2, -\frac{17\pi}{6}), (-2, -\frac{29\pi}{6})$

Ex. Plote the point $(3, 30^\circ)$.

Sol. $P(3, 30^\circ)$, $P(3, -330^\circ)$,

$P(-3, -150^\circ)$, $P(-3, 210^\circ)$

This the polar Coordinates of the point $P(3, 30^\circ)$

Remark. $\cos(\theta + 180^\circ) = -\cos\theta$.

$\sin(\theta + 180^\circ) = -\sin\theta$.

Ex. Plote the Following points

1. $P(5, 45^\circ)$ 2. $P(4, 90^\circ)$ 3. $P(6, 270^\circ)$.

Ex. Find all the Polar Coordinate of the point $P(2, 30^\circ)$.

Sol. 1. when $r = 2$.

The angle $\theta = 30 \mp 1 \cdot 360^\circ$

$30 \mp 2 \cdot 360^\circ$

$30 \mp 3 \cdot 360$

$\vdots$

Therefore the polar Coordinates $(2, 30 \mp n \cdot 360^\circ)$, $n = 0, 1, 2, 3, \dots$

or $(2, 30 + n \cdot 360^\circ)$, $n = 0, \mp 1, \mp 2, \mp 3, \dots$

2. For $r = -2$

The angle $\theta = -150 \mp 1 \cdot 360^\circ$

$= -150^\circ \mp 2 \cdot 360^\circ$

$= -150^\circ \mp 3 \cdot 360^\circ$

$\vdots$

Therefore the polar coordinates $(-2, -150^\circ \mp n \cdot 360^\circ)$, $n = 0, 1, 2, \dots$

or $(-2, -150^\circ + n \cdot 360^\circ)$, $n = 0, \mp 1, \mp 2, \dots$

$\therefore$ the polar coordinates are $(2, 30^\circ + n \cdot 360^\circ)$, $n = 0, \mp 1, \mp 2, \ldots$

$(-2, -150^\circ + n \cdot 360^\circ)$, $n = 0, \mp 1, \mp 2, \ldots$

Radian measure :-

$P(2, \frac{\pi}{6})$

IF we measure angle in radian the Formula that correspond $(2, \frac{\pi}{6} + 2n\pi)$, $n = 0, \mp 1, \mp 2, \ldots$

$(-2, -\frac{5\pi}{6} + 2n\pi)$, $n = 0, \mp 1, \mp 2, \ldots$

when $n = 0 \Rightarrow$ the Formular give $(2, \frac{\pi}{6})$, $(-2, -\frac{5\pi}{6})$

$n = 1 \Rightarrow$ " " " $(2, \frac{13\pi}{6})$, $(-2, \frac{7\pi}{6})$.