Remark :- To Find the cartesion Coordinate equivalent to Pola Coordinate and Vice verse, We use the Following eq:-

والعكس صحيح

- $x = r\cos\theta \Rightarrow \cos\theta = \frac{x}{r} \Rightarrow \theta = \cos^{-1}(\frac{x}{r})$.
- $y = r\sin\theta \Rightarrow \sin\theta = \frac{y}{r} \Rightarrow \theta = \sin^{-1}(\frac{y}{r})$.
- $x^2 + y^2 = r^2$, $\tan\theta = \frac{y}{x}$.

Ex 1 :- Find the Cartesion coordinate of the Following points

1. $P(2, 60^\circ)$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 2\cos 60^\circ$, $y = 2\sin 60^\circ$

$x = 2 \cdot \frac{1}{2}$, $y = 2 \cdot \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\Rightarrow x = 1$, $y = \sqrt{3}$

$\Rightarrow (x, y) = (1, \sqrt{3})$

2. $P(2, 90^\circ)$

$x = 2\cos 90^\circ$, $y = 2\sin 90^\circ$

$x = 2 \cdot 0$, $y = 2 \cdot 1$

$\Rightarrow (x, y) = (0, 2)$.

3. $P(-2, 0^\circ)$

$x = -2\cos 0^\circ$, $y = -2\sin 0^\circ$

$\cos 0 = 1$
$\sin 0 = 0$

$x = -2$, $y = 0$

$\Rightarrow (x, y) = (-2, 0)$.

4. $P(-2, 90^\circ)$ 5. $P(2, 0^\circ)$ Exe.

6. $P(2, \frac{\pi}{6})$ 7. $P(-4, \pi/3)$.

Ex2:- Find the Cartesian equivalent of the polar Coordinates.

1. $r^2\cos^2\theta - r^2\sin^2\theta = 1$

$\Rightarrow x^2 - y^2 = 1$.

2. $r^2\cos\theta\sin\theta = 4 \Rightarrow r\cos\theta \, r\sin\theta = 4 \Rightarrow xy = 4$.

3. $r\cos\theta = 2 \Rightarrow \boxed{x = 2}$ the graph is the vertical Line through $x = 2$

$r\sin\theta = 5 \Rightarrow y = 5$ " " " " horesential Line through $y = 5$

$r\sin\theta = -5 \Rightarrow y = -5$ is the horesential Line through $y = -5$

4. $7r\sin\theta + 3r\cos\theta = 12 \Rightarrow 7y + 5x = 12 \Rightarrow y = \frac{-5}{7}x + \frac{12}{7}$

The graph is the Line with slope $-\frac{5}{7}$, y-intercept $\frac{12}{7}$.

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x \Rightarrow x^2 - 4x + y^2 = 0$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4$

$\Rightarrow (x-2)^2 + y^2 = 4.$

The graph is the circle with $c(2,0)$, $r = 2$.

6. $r = 6(\sin\theta + \cos\theta)$ (Exe.)

7. $r = \dfrac{6}{\sqrt{9 - 5\sin^2\theta}}$ (Exe). بالتربيع

8. $r\sin\theta = e^{r\cos\theta}$. (Exc).

sol/ $\boxed{y = e^x}$

Examples:

1. Find all polar coordinates of the following points

1. $P(2, \frac{\pi}{4})$ 2. $P(4, \frac{\pi}{2})$ 3. $P(-5, -\frac{\pi}{4})$.

Solve ① $P(2, \frac{\pi}{4} + 2n\pi)$, $n = 0, \mp1, \mp2, \dots$

$P(-2, -\frac{3\pi}{4} + 2n\pi)$, $n = 0, \mp1, \mp2, \dots$

Solve ② $P(4, \frac{\pi}{2} + 2n\pi)$, $n = 0, \mp1, \mp2, \dots$

$P(-4, -\frac{\pi}{2} + 2n\pi)$, $n = 0, \mp1, \mp2, \dots$

③ Exc. ⟶

② Show that the point $(2, \frac{\pi}{2})$ lie on the curve $r = 2\cos 2\theta$

sol $r = 2\cos 2\theta$.

$\Rightarrow 2 = 2\cos 2(\frac{\pi}{2})$ $\cos\pi = -1$

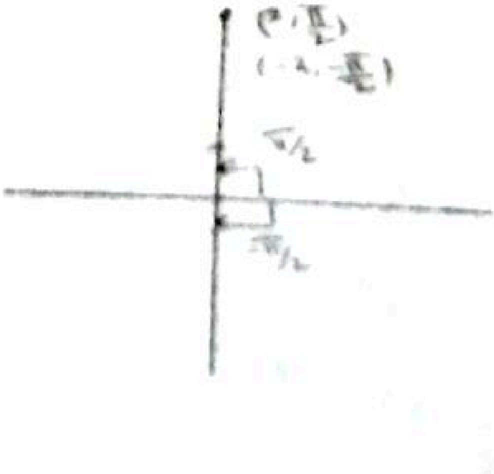
$\Rightarrow 2 \neq -2$. ∴ يمكن إيجاد نقطة للتحقق $P(r, \theta) = P(-r, \theta)$

But the point $(-2, -\pi/2)$ lie on the curve $r = 2\cos 2\theta$.

Since $-2 = 2\cos 2(-\frac{\pi}{2})$

$-2 = 2\cos(-\pi)$ $\cos -\pi = \cos\pi$

$\Rightarrow -2 = -2$ ⟵



Graphs of Polar equations:-

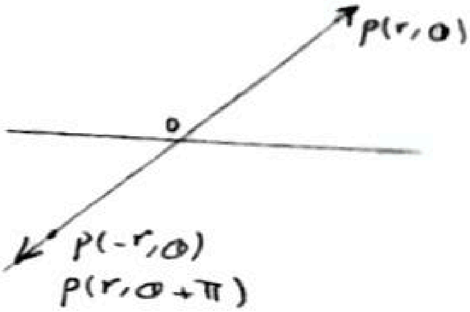
Symmetry

1. Symmetry about the origin

IF the equation is unchanged when r is replaced by $-r$ when θ is replaced by $\theta + \pi$

i.e $f(r, \theta) = f(-r, \theta)$

or $f(r, \theta) = f(r, \pi + \theta)$.

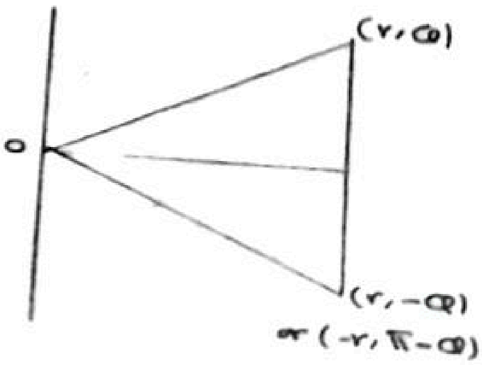


2. Symmetry about x-axis

IF the equation is unchanged when θ is replaced by $-\theta$ or the pair (r, θ) by the pair $(-r, \pi - \theta)$

i.e $f(r, \theta) = f(r, -\theta)$

or $f(r, \theta) = f(-r, \pi - \theta)$.



3. Symmetric about y-axis

IF the equation is unchanged when θ is replaced by $\pi - \theta$ or the pair (r, θ) by the pair $(-r, -\theta)$

i.e $f(r, \theta) = f(-r, -\theta)$

or $f(r, \theta) = f(r, \pi - \theta)$.