**Matrix Inversion: A Computational Algebra Approach**

**A standard method for computing A-1 for an r x n matrix A is to use Gauss-Jordan row operations to reduce the n x 2n augmented matrix [A | I] to [I | C], whence A-1 = C.**

**Instead, we propose reducing the [n by (n + 1)] augmented matrix [A | R], where R is an [n by 1] column vector of unspecified variable r. The G-J symbolic reduction of [A | R] to [I | D] yields A-1 as the matrix of coefficients of the ri's in D.**

**It is shown that, this approach yields substantial savings in both space and computational time over approaches used in existing computer algebra systems. A computational complexity analysis of the required number of operations shows that, for large n, their proposed method saves about 25% over the conventional method [Arsham 1993]. This reduction results from starting each row of R with just one (symbolic) term, instead of the n (numerical) terms in each row of I. Clearly, this approach has pedagogical advantages of their approach. One advantage is that simultaneously the generic linear system with coefficient matrix A is solved and A-1 is found, as discussed before.**

**Suppose we wish to find the inverse of the following matrix (if it exists):**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **2** | **1** |
| **A** | **=** |  |  |
|  |  | **1** | **-1** |

|  |
| --- |
|  |
| **2** |  |  | **1** |  |  |  |  | **r1** |
| **1** |  |  | **-1** |  |  |  |  | **r2** |

|  |
| --- |
|  |
| **1** | **1/2** |  |  | **r1 / 2** |
| **0** | **-3/2** |  |  | **-r1 / 2 + r2** |

|  |
| --- |
|  |
| **1** |  |  | **0** |  |  |  |  | **r1 / 3 + r2 / 3** |
| **0** |  |  | **1** |  |  |  |  | **r1 / 3 - 2r2 / 3** |

**Therefore the inverse matrix is:**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **1/3** | **1/3** |
| **A-1** | **=** |  |  |
|  |  | **1/3** | **-2/3** |

**Notice: A matrix possessing an inverse is called nonsingular, or invertible. A matrix is called singular if it does not have an inverse. For example, the following matrix is a singular one:**

**1    6    4
 2    4   -1
-1    2    5**

**Therefore, in applying the above procedure for inverting a matrix, if the matrix is a singular one, then at least one row will have All Zero Elements during the G-J operations.**

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**Learning Objects: Online Interactive Solvers**

**Computer Assisted Learning: I do recommend the following online interactive solvers sites to understand the concepts presented on this site by performing some numerical experimentation:**

[**System of Equations and Matrix Inversion**](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/SysEq.htm)**.**[**Linear Optimization**](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/LPTools.htm)**.**

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**Rounding Errors and Stability Analysis**

**Rounding errors is due to the hardware limitation of any computer package. Therefore, one must be concerned about the stability analysis of the solution with respect to the input parameters. Following are three numerical examples for Linear programming, matrix inversion, and solving system of linear equations, respectively:**

**Linear Programming: Consider the following problem:**

**Max 6X1 + 4X2
subject to:
3.01X1 + 2X2 £ 24
X1 + 2X2 £ 16 all decision variables ³ 0.**

**The optimal solution is (X1 = 3.9801, X2 = 6.0100). However, the optimal solution for the same problem but with a slight change in the constraints' coefficients matrix one may get a completely different optimal solution. For example, if we change 3.01 to 2.99, then the optimal solution is (X1 = 8.0268, X2 = 0). That is, decreasing the one element of the technology-matrix by 0.66%, the solution changes drastically! Therefore, the optimal solution is unstable with respect to this input parameter.**

**Matrix Inversion: Consider the following matrix:**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **1.9998** | **0.9999** |
| **A** | **=** |  |  |
|  |  | **5.9994** | **3.0009** |

**This matrix has a proper inverse, however rounding the elements of matrix A,**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **2** | **1** |
| **Ar** | **=** |  |  |
|  |  | **6** | **3** |

**it becomes a singular matrix, i.e., it has no inverse.**

**Solving System of Equations: Consider the following system of equation:**

**2.04X1 + 2.49X2 = 0.45
2.49X1 + 3.04X2 = 0.55**

**which has a unique solution (X1 = -1, X2 = 1). However, rounding the coefficient matrix to:**

**2.0X1 + 2.5X2 = 0.45
2.5X1 + 3.0X2 = 0.55**

**now the solution become (X1 = 0.1, X2 = 0.1) a startling change.**