• <u>trigonometric functions</u>

trigonometric functions can be defined using a right-angled triangle, but they are understood through the unit circle, which is a circle with a radius of 1 centered at the origin of the coordinate plane.

يمكن تعريف الدوال المثلثية باستخدام مثلث قائم الزاوية، ولكن يتم فهمها من خلال دائرة الوحدة وهي دائرها نصف قطرها واحد ومركزها أصل المستوى الاحداثي.

For any angle θ, the point on the unit circle corresponding to the angle has coordinates (x,y), where:

- $x = \cos(\theta)$
- $y = \sin(\theta)$
- The angle θ is measured counterclockwise from the positive x-axis.

يتم قياس الزاوية θ عكس اتجاه عقارب الساعة من المحور السيني الموجب

Then,

1- $Sin(\theta) = \frac{y}{r}$

2- $cos(\theta) = \frac{x}{r}$

3- $tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{y}{x}$,

4- $cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{x}{y}$

5- $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x}$

6- $csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y}$

Remarks:

1- (القياس الستيني) $180^\circ = \pi$ radians (القياس القطري)

2- To change radians to degree, multiply radians by $\frac{180^\circ}{\pi}$.

3- To change degree to radians, multiply degree by $\frac{\pi}{180^\circ}$.

Example:

1- Change $30°, 45°, 60°, 90°$ to radians?

2- Change $\frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{3}$ to degree?

Sol:

1- $30° \times \frac{\pi}{180°} = \frac{\pi}{6}$ red.

2- $\frac{\pi}{6} \times \frac{180°}{\pi} = 30°$

Remark:

1- $sin^2(\theta) + cos^2(\theta) = 1$

2- $1 + tan^2(\theta) = sec^2(\theta)$

3- $1 + cot^2(\theta) = csc^2(\theta)$

4- $sin^2(\theta) = \frac{1-\cos(\theta)}{2}$

5- $cos^2(\theta) = \frac{1+\cos(\theta)}{2}$

6- $\sin(\theta + 2\pi) = \sin(\theta)$

7- $\cos(\theta + 2\pi) = \cos(\theta)$

8- $Sin(-\theta) = -sin(\theta)$

9- $cos(-\theta) = cos(\theta)$

Graph of Trigonometric Function

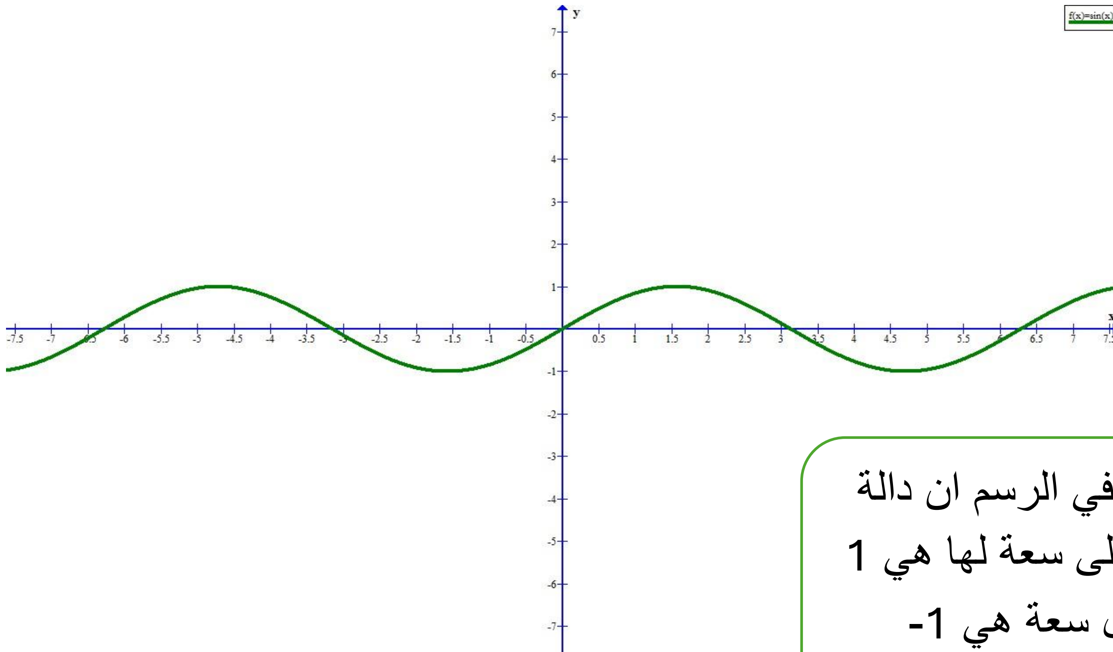
1. $y = \sin(\theta)$

θ	sin	cos	tan
0	0	1	0
30	1/2	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90	1	0	∞
180	0	-1	0
270	-1	0	∞
360	0	1	0

هذا الجدول يحتوي اهم قيم
للدالتين sin , cos التي
نحتاجها في الرسم

لرسم دالة sin

$X=\theta$	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0
$-\pi/2$	-1
$-\pi$	0
$-3\pi/2$	1
-2π	0

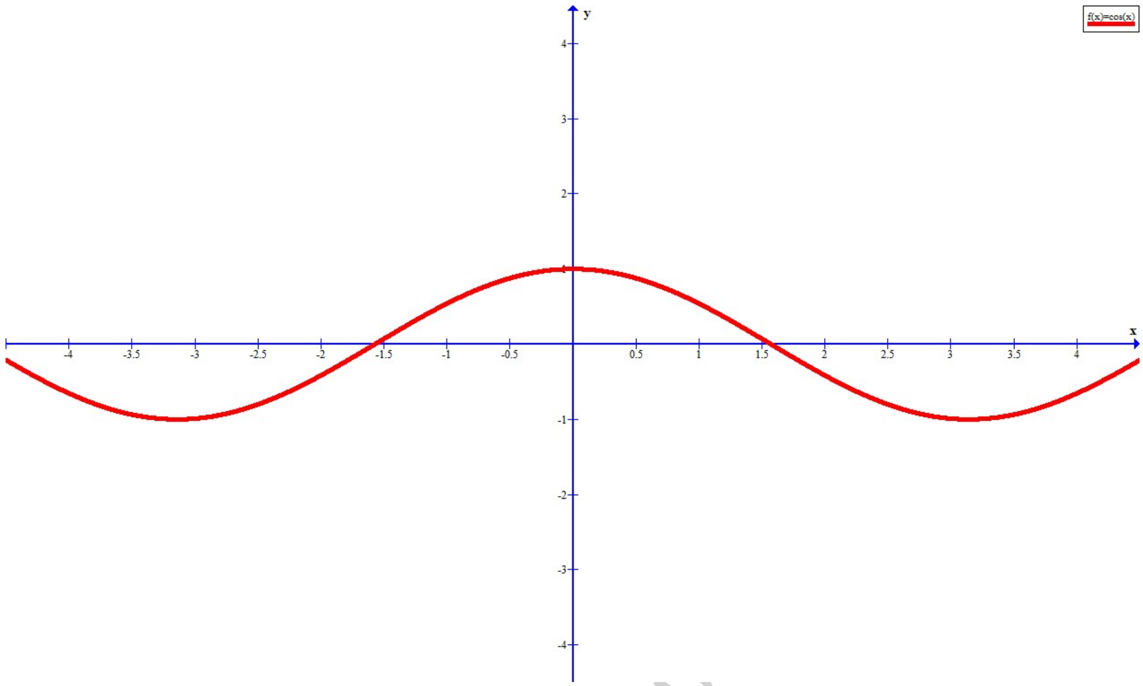


نلاحظ في الرسم ان دالة sin اعلى سعة لها هي 1 واقل سعة هي 1-

$2. y = cos(\theta)$

لرسم دالة cos

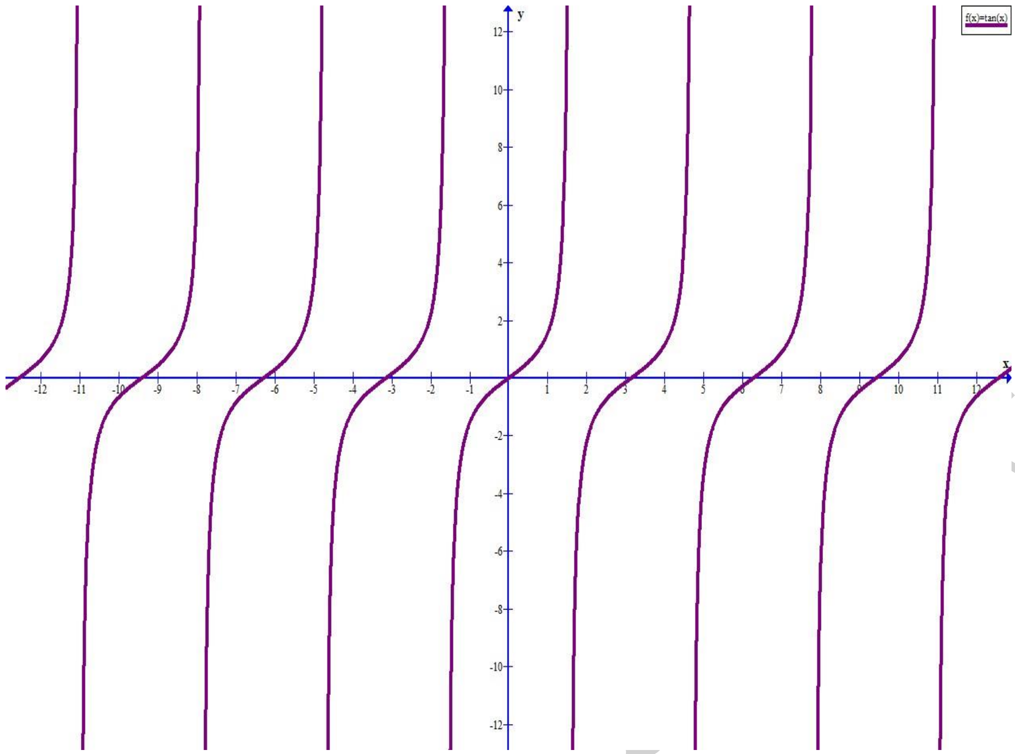
$X=\theta$	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1
$-\pi/2$	0
$-\pi$	-1
$-3\pi/2$	0
-2π	1



3. $y = tan(\theta)$

$$\tan(\theta) = \frac{sin\theta}{cos\theta}$$

X=θ	y
0	0
$\pi/2$	∞
π	0
$3\pi/2$	∞
2π	0
$-\pi/2$	∞
$-\pi$	0
$-3\pi/2$	∞
-2π	0



H.W

1. $y = cot(\theta)$
2. $y = csc(\theta)$
3. $y = sec(\theta)$

Composition of functions

The composition of functions involves combining two functions to form a new function. If you have two functions $f(x)$ and $g(x)$, the composition of f and g is written as $(f \circ g)(x)$, which means you apply $g(x)$ first and then apply $f(x)$ to the result of g(x).

<u>Definition:</u>

Given two functions f and g, the composition $(f \circ g)(x)$

Is defined as:

$$(f \circ g)(x) = f\big(g(x)\big)$$

This means you first evaluate $g(x)$ and then substitute that result into the function f .

for example,

$f(x) = 2x + 1$ and $g(x) = x^2$

Now, we want to find $(f \circ g)(x)$. This means we will first apply $g(x)$, and then apply $f(x)$ to the result of result of $g(x)$.

1-Evaluate $g(x)$: $g(x) = x^2$.

2-Now apply $f(x)$ to $g(x) = x^2$: $f\big(g(x)\big) = f(x^2) = 2x^2 + 1$.

So, the composition $(f \circ g)(x)$ is:

$$(f \circ g)(x) = 2x^2 + 1$$

<u>Remark:</u>

1- Order matters: $(f \circ g)(x)$ is not necessary the same as $(g \circ f)(x)$.

Generally, $f\big(g(x)\big) \neq g\big(f(x)\big)$.

For example,

Let's consider the following functions:

$$f(x)2x, \quad g(x) = x + 3$$

Now, let's compute both compositions:

$$(f \circ g)(x) = f\big(g(x)\big)$$

First, evaluate $g(x)$:

$$g(x) = x + 3$$

Now apply, f to $g(x)$:

$$f\big(g(x)\big) = f(x + 3) = 2(x + 3) = 2x + 6$$

So, $(f \circ g)(x) = 2x + 6$

$$(g \circ f)(x) = g\big(f(x)\big)$$

First, evaluate $f(x)$:

$$f(x) = 2x$$

Now apply g to $f(x)$:

$$g\big(f(x)\big) = g(2x) = 2x + 3$$

So, $(g \circ f)(x) = 2x + 3$.

As you can see, $f\big(g(x)\big) \neq g\big(f(x)\big)$, which show that the order matters in function composition.

2- Associativity:

for function composition to be associative, it means that if we have three functions f, g, and h.

$$\big(f \circ (g \circ h)\big) = ((f \circ g) \circ h)$$

This implies that if we first compose g and h, and then compose the result with f, it's the same as first composing f and g, and then composing that result with h.

For example,

Consider three functions $f(x) = 2x, g(x) = x + 1,$ and $h(x) = x^2$.

Let's check the associativity of composition:

First find $\big(f \circ (g \circ h)\big)$

$$g \circ h(x) = g\big(h(x)\big) = g(x^2) = x^2 + 1$$

Now,$f \circ (\quad)(xg \circ h) = f(x^2 + 1) = 2(x^2 + 1) = 2x^2 + 2$

Second find $((f \circ g) \circ h)$

$$f \circ g(x) = f(g(x)) = f(x + 1) = 2(x + 1) = 2x + 2$$

Now, $(f \circ g) \circ h(x) = (2x + 2) \circ h = 2(x^2) + 2 = 2x^2 + 2$

Since both approaches give the same result $2x^2 + 2$, this demonstrates that composition is associative.

<u>H.W</u>

1- $f(x) = x + 1$ and $g(x) = 2x$,find $f \circ g(5)$.

2- $f(x) = \sqrt{1 - x}$ and $g(x) = \sqrt{x - 1}$
find $g \circ f(1 - a)$.

3- $f(x) = x^2$d $g(x) = 2x + 3$,find $f \circ g(1 + r)$,

4- $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{x-1}{x}$,find $f \circ g(7)$.

5- Let $f(x) = 2x + 1$,find g(x) such that
$f \circ g(x) = x^2$