

الغايات *Limits*

هو مفهوم يستخدم لدراسة سلوك الدوال أو المتغيرات عندما تقترب من قيمة معينة ، سواء كانت قيمة حقيقية او قيمة لا نهائية.

Definition:

a limit describes the value that a function approaches as the input (independent variable), For example, if we have a function $f(x)$, we are interested in the value that $f(x)$ gets closer to as x approaches a certain value a .

This is written as:

$$\lim_{x \rightarrow a} f(x) = L$$

This means "as x approaches a , $f(x)$ approaches the value L ."

وهذا يعني أن "عندما يقترب x من a ، تقترب $f(x)$ من القيمة L "

Types of Limits:

- Finite Limit: A limit is finite if the function approaches a specific, finite value as x approaches a .

$$\lim_{x \rightarrow a} f(x) = L$$

For example,

Consider the function $f(x) = 3x + 1$. find the limits of this function when $x \rightarrow 2$.

$$\lim_{x \rightarrow 2} (3x + 1)$$

To find this limit,

we simply substitute $x = 2$ into the function

$$f(2) = 3(2) + 1 = 7$$

thus,

$$\lim_{x \rightarrow 2} (3x + 1) = 7$$

- Infinite Limit: A limit is infinite if the function approaches infinity (either positive or negative) as x approaches a .

$$\lim_{x \rightarrow a} f(x) = \infty, \quad \lim_{x \rightarrow a} f(x) = -\infty$$

For example,

Consider the function $f(x) = \frac{1}{x}$, Let's find the limit as

$x \rightarrow 0^+$:

$$\lim_{x \rightarrow 0^+} \frac{1}{x}$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$$

- Limit at Infinity: A limit at infinity happens when x approaches infinity (positive or negative).

$$\lim_{x \rightarrow \infty^+} f(x) \quad , \quad \lim_{x \rightarrow \infty^-} f(x)$$

For example,

Consider the function $f(x) = \frac{1}{x}$, Let's find the limit as

$$x \rightarrow \infty$$

$$\lim_{x \rightarrow \infty} \frac{1}{x}$$

As x increases without bound

Therefore,

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

عندما تقترب x من ∞ بالاتجاه الموجب فإن $f(x) = \frac{1}{x}$ سوف تقترب من الصفر لان المقام ونفس الحالة عندما تقترب x من ∞ من الاتجاه السالب. وعليه في كلا الحالتين تقترب الدالة من الصفر

Example: find

$$\lim_{x \rightarrow \infty} \frac{1}{x^3}$$

Solution/

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{1}{x^3} &= \lim_{x \rightarrow \infty} \left(\frac{1}{x} \cdot \frac{1}{x} \cdot \frac{1}{x} \right) \\ &= \left(\lim_{x \rightarrow \infty} \frac{1}{x} \right) \cdot \left(\lim_{x \rightarrow \infty} \frac{1}{x} \right) \cdot \left(\lim_{x \rightarrow \infty} \frac{1}{x} \right) \\ &= 0.0.0 = 0\end{aligned}$$

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$$\lim_{x \rightarrow \infty} \frac{1}{x^3} = 0$$

Example: find

$$\lim_{x \rightarrow \infty} \frac{2x^3 + 1}{5x^3 + x^2 - 1}$$

Solution/

$$\lim_{x \rightarrow \infty} \frac{\frac{2x^3}{x^3} + \frac{1}{x^3}}{5 \frac{x^3}{x^3} + \frac{x^2}{x^3} - \frac{1}{x^3}}$$

نقسم البسط والمقام
على المتغير الذي
يحمل أعلى أس
في هذا المثال
أعلى أس هو x^3

$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x^3}}{5 + \frac{1}{x} - \frac{1}{x^3}} \\
 &= \frac{\lim_{x \rightarrow \infty} (2) + \lim_{x \rightarrow \infty} \left(\frac{1}{x^3}\right)}{\lim_{x \rightarrow \infty} (5) + \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right) - \lim_{x \rightarrow \infty} \left(\frac{1}{x^3}\right)} \\
 &= \frac{\lim_{x \rightarrow \infty} (2) + 0}{\lim_{x \rightarrow \infty} (5) + 0 - 0} = \frac{2}{5}
 \end{aligned}$$

Example:

Consider the function $f(x) = \frac{x^2 - 4}{x - 2}$. Let's find the limit of this function as $x \rightarrow 2$.

Solution/

$$\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right) = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{(x - 2)}$$

$$\lim_{x \rightarrow 2} (x + 2) = 4$$

Homework: find

$$1. \lim_{x \rightarrow -\infty} \frac{x^5 - x^4 + x^2 - 2}{4x^5 + x^3 - x + 3}$$

$$2. \lim_{x \rightarrow \infty} \frac{x^4 + x^2 - x + 1}{x^5 + x^3 - 2x + 3}$$

$$3. \lim_{x \rightarrow 10} (x^2 + x - 2)$$

$$4. \lim_{x \rightarrow 2} [5(2x - 4)(x - 1)^2]$$

$$5. \lim_{t \rightarrow 3} \left(\frac{t^3 - 1}{t - 1} \right)$$

$$6. \lim_{x \rightarrow -\infty} \left(\frac{1}{x} \right)$$

$$7. \lim_{x \rightarrow} \left(\frac{x^2 - 25}{x - 5} \right)$$

$$8. \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+5} - \sqrt{5}}{x} \right)$$