Chapter three

The natural logarithm and exponential Function and L' Hopital Rule

1- The natural logarithmic Function دالة اللوغارتم الطبيعي

The natural logarithm denoted by $\mathbf{ln}(\boldsymbol{x})$

<u>Properties of $\ln(x)$:</u>

1- $D_f = \{x : x > 0\} = R^+/\{0\}$

2- $R_f = \{-\infty < y > 0\} = R\}$

3-$\ln(1) = 0$

4-$\ln(e) = 1$ where e=2.7182

5- $\ln(a.b) = \ln(a) + \ln(b)$

6-$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

7-$\ln a^r = rln(a) \quad r \in R \ , \quad a \in R^+$

8- $\ln\left(\frac{1}{x}\right) = -\ln x$

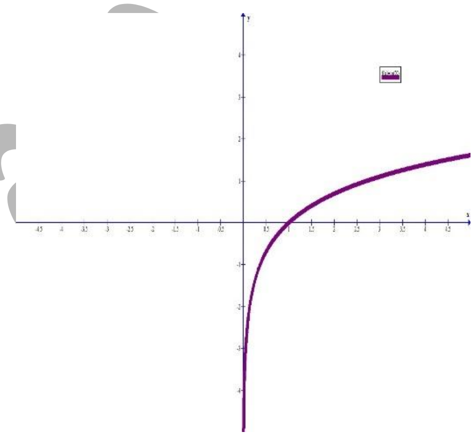
9-$\ln(x) > 0$ if $x > 0$

10- $\ln(x) < 0$ if $x < 0$

11- $\lim_{x \to 0} \ln x = -\infty$

12- $\lim_{x \to \infty} \ln x = \infty$

The graph of $\ln x$



1- $y = \ln x$

$\Rightarrow \quad D_y = R^+ \quad , R_y = R$

2- $\ln 1 = 0$ (0,1) نقطة التقاطع

3- $\lim_{x \to 0} \ln x = -\infty$

4- $\lim_{x \to \infty} \ln x = \infty$

Derivative of $\ln x$

$$y = \ln x \ \rightarrow \ y' = \frac{1}{x} \ .1$$

Or

$y = \ln u$ where u is function

$$y' = \frac{1}{u} \ .du$$

Example:

1- let $y = \ln x^2 \Rightarrow y' = \frac{1}{x^2} \ (2x) = \frac{2}{x}$

2- let $y = \ln(\sin x) \Rightarrow y' = \frac{1}{\sin x} \ (\cos x) = \cot x$

3- let $y = [\ln(x^2 + 2)]^5 \Rightarrow y' = 5 \ [\ln(x^2 + 2)]^4 \ (\frac{1}{x^2+2})(2x)$

2- The exponential Function الدالة الأسية

هي معكوس دالة اللوغاريتم الطبيعي ويرمز لها بالرمز exp or e^x

Properties of $\exp(x)$

1- $y = e^x \Rightarrow \quad x = \ln y$

2- $D_y = R \qquad , R_y = R^+$

3- $e^0 = 1$

4- $e^a . e^b = e^{a+b}$

5- $\frac{e^a}{e^b} = e^{a-b}$

6- $(e^a)^r = e^{ar}$

7- $e^{-a} = \frac{1}{e^a}$

8- $\ln e^x = x \qquad \forall x > 0$

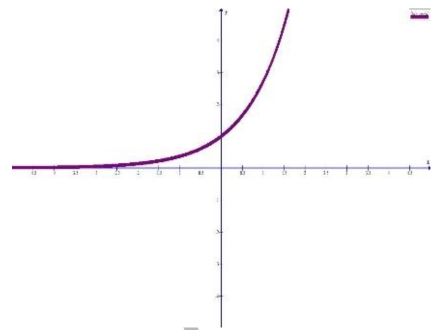
9- $\lim_{x \to \infty} e^x = e^{\infty} = \infty$

10- $\lim_{x \to -\infty} e^x = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

The graph of e^x:

1- $y = e^x$

$$\Rightarrow \quad D_y = R \quad , R_y = R^+$$



2- $e^0 = 1$ (1,0) نقطة التقاطع

3- $\lim_{x \to \infty} e^x = e^{\infty} = \infty$

4- $\lim_{x \to -\infty} e^x = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

Derivative of e^x:

Let $y = e^x \quad \Rightarrow y' = e^x \,.1$

Or

If $y = e^u$ where u is a function, then

$$y' = e^u \,.du$$

Example:

Find y' of the following function

1- $y = e^{\tan x} \Rightarrow y' = e^{\tan x}.(sec^2 x).1$

2- $y = e^{x^2+\sin x} \Rightarrow y' = e^{x^2+sinx}.(2x + \cos x)$

3-$y = \sin(e^{x^2}) \Rightarrow y' = \cos(e^{x^2})[e^{x^2}(2x)]$

$$= (2x)(e^{x^2}).[\cos(e^{x^2})]$$

Example:

Find $\grave{y}$ y'' y''' of $y = e^{x+1}$ and $y'''(-1)$

Sol:

$\grave{y} = e^{x+1} . (1) = e^{x+1}$

$y'' = e^{x+1}(1) = e^{x+1}$

$y''' = e^{x+1}(1) = e^{x+1}$, $y'''(-1) = e^{-1+1} = e^{0} = 1$

H.w

find $\grave{y}$ for each the following function

1- $y = \sin(\ln x)$

2- $y = \cos(e^{x})$

3- $y = \frac{e^{x}}{\sin x + 1}$

4- $y = \frac{\sin x + \sec x}{\ln x}$

5- $y = e^{\tan x + \sqrt{x}}$

6- $y = e^{\sin x + \ln x}$