Definition 1.3

Two events F_1 and F_2 are said to be "disjoint" or "mutually exclusive" if $F_1 \cap F_2 = \phi$.

Definition 1.4

$n -$ factorial: It can find $n -$ factorial by the following formula

$$n! = n(n-1)(n-2) \dots 2.1 = n(n-1)! \, .$$

Note

$0! = 1$,

$1! = 1.$

Definition 1.5 Permutations

A permutation is an arrangement of all or part of a set of objects.

Note 1

The number of permutations of n distinct objects is

$$\boldsymbol{n}!.$$

Note 2

The number of permutations of n distinct objects taken r at a time is

$$P_r^n = \frac{n!}{(n-r)!}.$$

Note 3

The number of permutations of n distinct objects arranged in a circle is

$$(n-1)!.$$

Note 4

The number of distinct permutations of n things of which n_1 are of one kind , n_2 of a second kind,…, n_k of a k^{th} kind is

$$\frac{n!}{n_1!\,n_2!\,...\,n_k!}.$$

Example 1

How many permutations of letters a , b , and c taken 2 at time?

Sol.

$$P_r^n = \frac{n!}{(n-r)!},$$

$$n = 3 \, , r = 2$$

$$\Rightarrow P_2^3 = \frac{3!}{(3-2)!} = 3! = 6 \, .$$

Example 2

How many permutations for 3 letters say, a , b and c ?

Sol.

$$n! = 3! = 3.2.1 = 6 \, .$$

Example 3

Consider the word **STATISTICS**, find number of permutation.

Sol.

$n = 10$, total number of characters.

$n_1 = 3$, number of letter S.

$n_2 = 3$, number of letter T.

$n_3 = 2$, number of letter I.

$n_4 = 1$, number of letter A.

$n_5 = 1$, number of letter C.

$$\Rightarrow \frac{n!}{n_1!\,n_2!\,n_3!\,n_4!\,n_5!} = \frac{10!}{3!\,3!\,2!\,1!\,1!}$$

$$= \frac{10!}{3!\,3!\,2!} = 50400.$$

Definition 1.6 Combination

A combination is a selection of objects made without regard to order.

A subset of r objects selected without regard to their order from a set of n different objects is called a combination of the n objects taken r at a time and denoted by C_r^n or $\binom{n}{r}$.

Note 1

The number of combination of a set of n different objects taken r at a time is

$$\binom{n}{r} = C_r^n = \frac{n!}{r!\,(n-r)!}.$$

Note 2

The relation between permutation and combination is

$$C_r^n = \frac{P_r^n}{r!} \quad \text{or} \quad P_r^n = r!\,C_r^n .$$

Note 3

$$\binom{n}{n-r} = \binom{n}{r}$$

Proof

$$\binom{n}{n-r} = \frac{n!}{(n-r)!\,[n-(n-r)]!}$$

$$= \frac{n!}{(n-r)!\,(n-n+r)!}$$

$$= \frac{n!}{(n-r)!\,r!} = \binom{n}{r} = C_r^n. \quad \blacksquare$$

Example

In how many ways can a committee consisting of 3 men and 2 women be chosen from 7 men and 5 women ?

Sol.

No. of ways of choosing the men is

$$\binom{7}{3} = C_3^7 = \frac{7!}{3! \times (7-3)!} = \frac{7!}{3! \times 4!} = \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!}$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35.$$

No. of ways of choosing the women is

$$\binom{5}{2} = C_2^5 = \frac{5!}{2! \times 3!} = \frac{5 \times 4 \times 3!}{2! \times 3!} = \frac{5 \times 4}{2 \times 1} = 10.$$

To choice a committee we have $10 \times 35 = 350$ ways.

Note

If $a + b = n$ then $\binom{n}{b} = \binom{n}{a}$.

Example

$$3\,,7 \Rightarrow \overbrace{3}^{a} + \overbrace{7}^{b} = \overbrace{10}^{n}$$

$$\binom{10}{7} = \frac{10!}{7! \times 3!} = \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1} = 120,$$

$$\binom{10}{3} = \frac{10!}{3! \times 7!} = \frac{10 \times 9 \times 8 \times 7!}{7! \times 3 \times 2 \times 1} = 120.$$