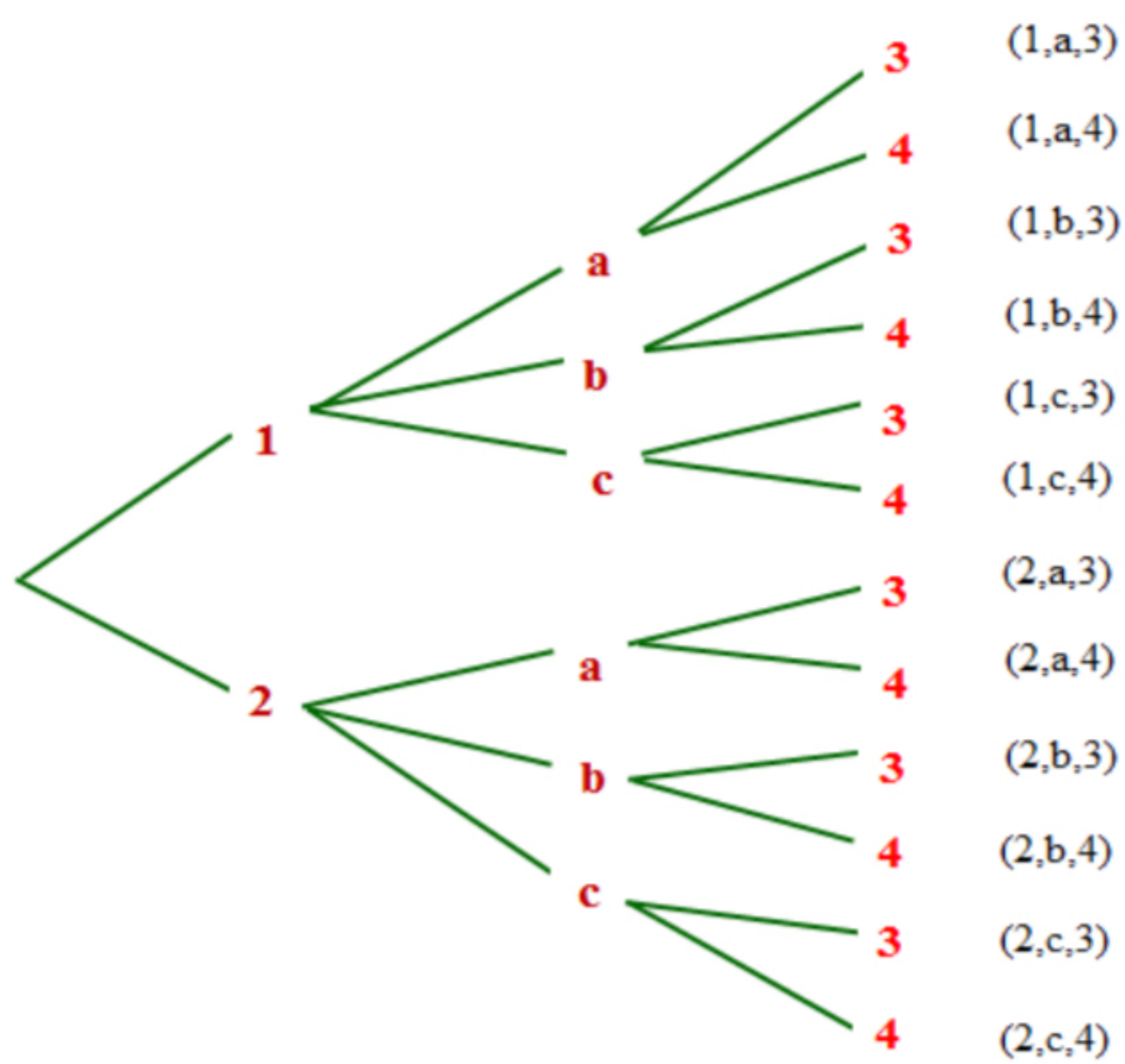
Tree Diagrams

Is a device used to enumerate all the logical possibilities of a sequence of events where each event can occur in a finite number of ways. The construction of tree diagrams is illustrated in the following example:

Example

Find the product set $A \times B \times C$ where $A = \{1,2\}$, $B = \{a, b, c\}$ and $C = \{3,4\}$.

Sol.



Observe that the tree is constructed from left to right, and that number of branches at each point corresponds to the number of ways the next event can occur.

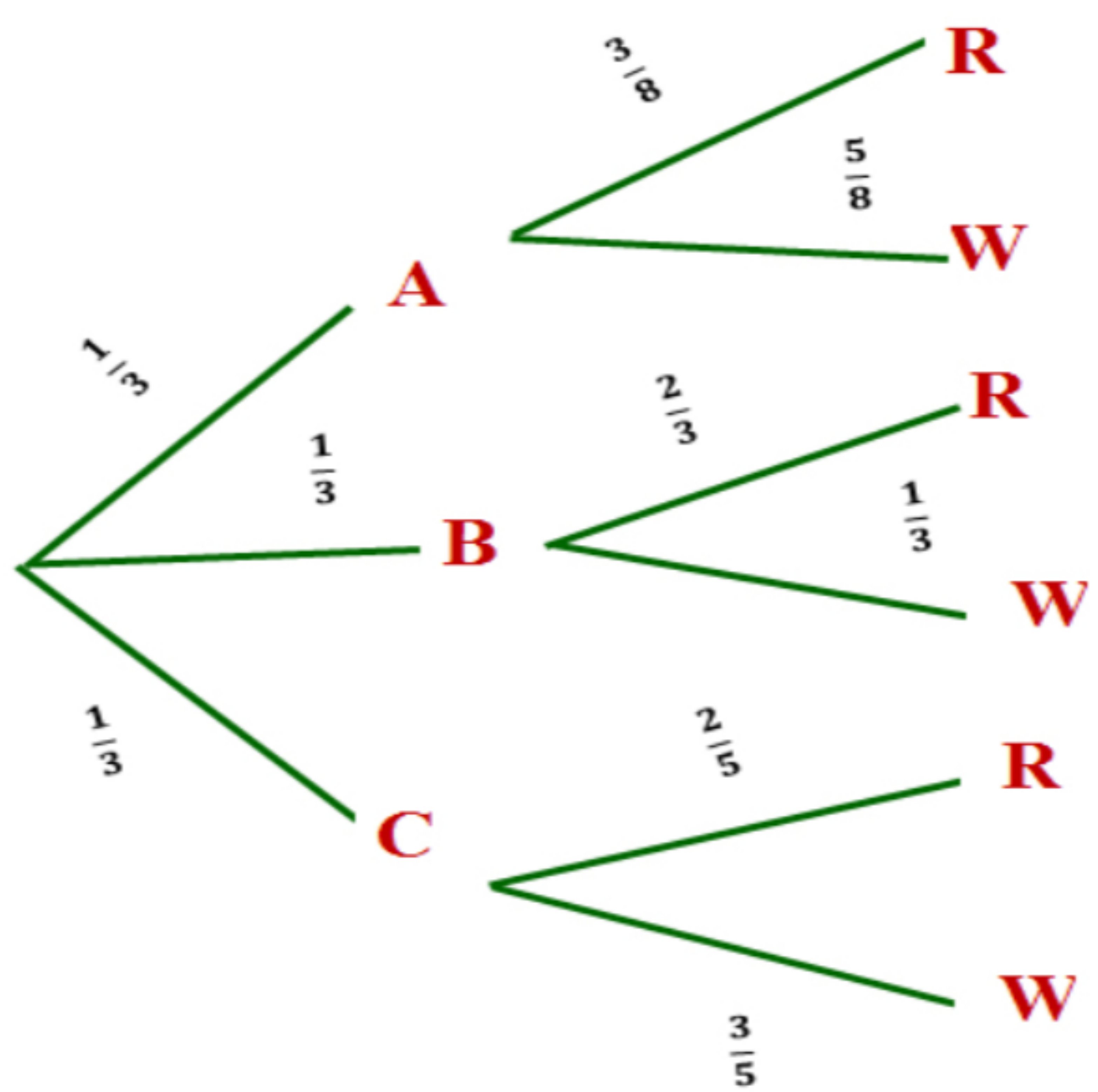
Example

We are given three urns as follows:

Urn A contains 3 red and 5 white marble. Urn B contains 2 red and 1 white marble. Urn C contains 2 red and 3 white marble. An urn is selected at random and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A ?

Sol.

Construct the tree diagram as shown in Figure below:



We seek the probability that A was selected, given that the marble is red, that is $P(A|R)$.

In order to find $P(A|R)$, it is necessary first to compute $P(A \cap R)$ and $P(R)$. The probability that urn A is selected and a red marble drawn is

$$\frac{1}{3} \times \frac{3}{8} = \frac{1}{8}, \text{ that is } P(A \cap R) = \frac{1}{8}.$$

Since there are three paths leading to a red marble

$$P(R) = \left(\frac{1}{3}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{5}\right) = \frac{173}{360}.$$

Thus

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{\frac{1}{8}}{\frac{173}{360}} = \frac{45}{173}.$$

Another method using Bayes' theorem

$$P(A|R) = \frac{P(A)P(R|A)}{P(A)P(R|A) + P(B)P(R|B) + P(C)P(R|C)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{3}{8}\right)}{\left(\frac{1}{3}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{5}\right)} = \frac{45}{173}.$$

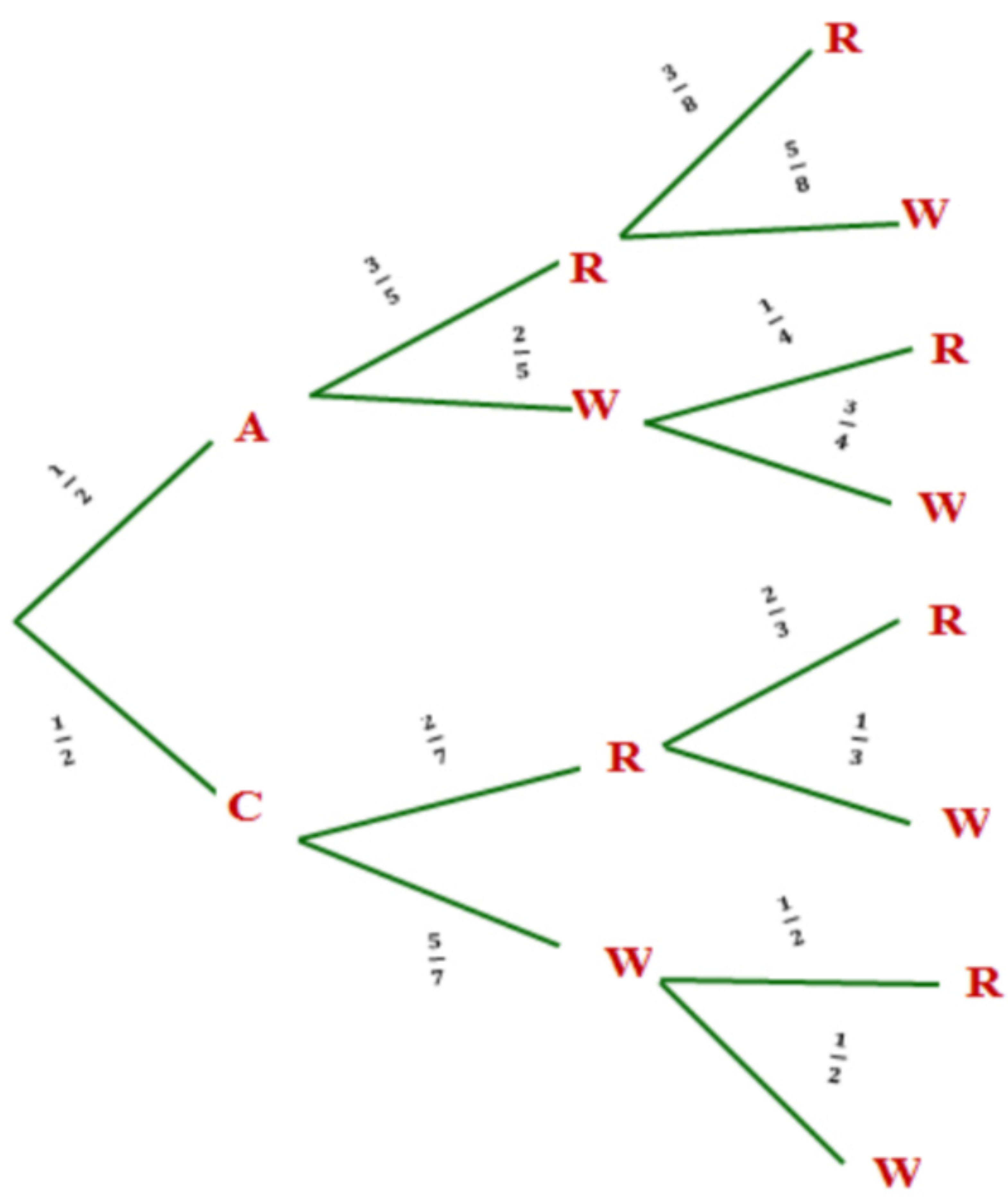
Example

We are given two urns as follows:

Urn A contains 3 red and 2 white balls. Urn B contains 2 red and 5 white balls. An urn is selected at random; a ball is drawn and put into the other urn, then a ball is drawn from the second urn. Find the probability (p) that both balls drawn are of the same color.

Solution

Construct the following tree diagram:



Note that if urn A is selected and a red ball drawn and put into urn B, then urn B has 3 red balls and 5 white balls.

Since there are four paths which leads to two balls of the same color

$$p = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{7}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{7}\right)\left(\frac{1}{2}\right) = \frac{901}{1680}.$$