

Solve equation chapter one

Q

1.4 If the refractive index for a piece of optical glass is 1.5250, calculate the speed of light in the glass. *Ans.* $1.9659 \times 10^8 \text{ m/s}$

Solution:

$$n = \frac{c}{v} = \frac{3 \times 10^8}{v} = 1.9672 * 10^8 \text{ m/s}$$

1.5 Calculate the difference between the speed of light in kilometers per second in a vacuum and the speed of light in air if the refractive index of air is 1.0002340. Use velocity values to seven significant figures.

Q1.5

Solution:

$$c = \frac{d}{t} = \frac{1.734 \text{ km}}{1/18000\text{s}} = 3.12 * 10^5 \text{ km /s}$$

$$C = 3 * 10^8 \text{ m/s}$$

$$n = \frac{c}{v}$$

$$1.000234 = \frac{3 * 10^5}{v}$$

$$C = 299931.01586635 = 2.99931015 \times 10^5 \text{ km/s}$$

1.6 If the moon's distance from the earth is $3.840 \times 10^5 \text{ km}$, how long will it take microwaves to travel from the earth to the moon and back again?

Q1.6

Solution

$$C = d/t$$

$$t = \frac{3.84 \times 10^5 \text{ km}}{3 \times 10^5 \text{ km/s}} \implies 2t = 1.28 \times 2 \text{ sec}$$

$$2t = 2.56 \text{ sec.}$$

Q1.7

Solution:

$$n = \frac{c}{v}$$

$$v = \frac{d}{t} = 1.5 \times 10^8 \text{ km}$$

$$t = \frac{d}{v} = \frac{1.5 \times 10^8}{3 \times 10^5} = 500 \text{ sec.}$$

$$1 \text{ min} = 60 \text{ sec.} \quad = 8 \text{ min } 20 \text{ sec.}$$

1.8 A beam of light passes through a block of glass 10.0 cm thick, then through water for a distance of 30.5 cm, and finally through another block of glass 5.0 cm thick. If the refractive index of both pieces of glass is 1.5250 and of water is 1.3330, find the total optical path.

Q1.8

Solution:

$$\text{Opl} = \Delta_1 + \Delta_2 + \Delta_3 = n_1 d_1 + n_2 d_2 + n_3 d_3$$

$$= 1.525 \times 10 \text{ cm} + 1.333 \times 30.5 + 5 \times 1.525 = 63.5315 \text{ cm}$$

1.9 A water tank is 62.0 cm long inside and has glass ends which are each 2.50 cm thick. If the refractive index of water is 1.3330 and of glass is 1.6240, find the overall optical path.

$$\text{Opt} = \Delta 1 + \Delta 2 + \Delta 3 = n_1 d_1 + n_2 d_2 + n_3 d_3$$

$$= 2.5 * 1.624 + 62 * 1.333 + 2.5 * 1.624$$

$$= 90.766$$

1.11 A ray of light in air is incident on the polished surface of a block of glass at an angle of 10° . (a) If the refractive index of the glass is 1.5258, find the angle of refraction to four significant figures. (b) Assuming the sines of the angles in Snell's law can be Replaced by the angles themselves, what would be the angle of refraction? (c) Find the percentage error.

Q1.11

Solution

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 * \sin 10^\circ = 1.5258 \sin \theta_2$$

$$\sin \theta_2 = 0.113807955$$

$$\theta_2 = 6.53487^\circ$$

$$\frac{\phi}{\phi'} = \frac{n'}{n}$$

b)

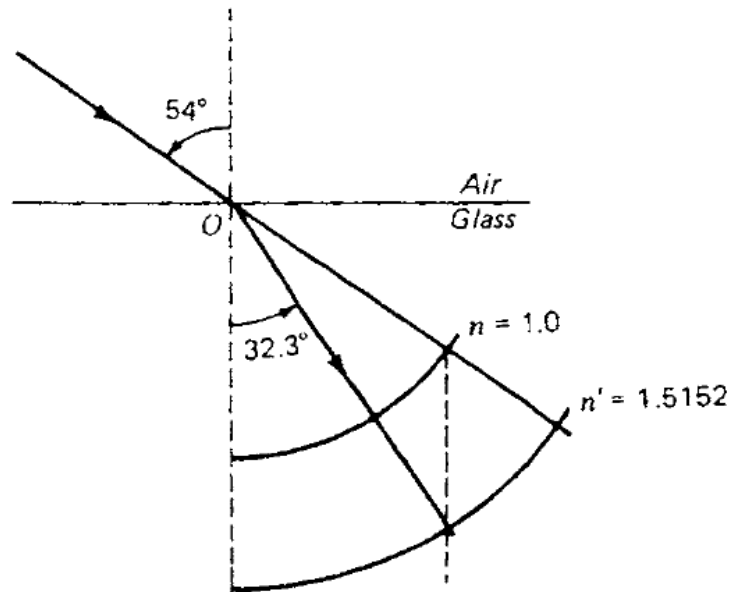
$$\frac{10}{\phi'} = \frac{1.5258}{1}$$

$$\phi' = 6.5539^\circ$$

$$\text{ERROE} = 100\% * \frac{6.5539 - 6.5348}{6.5539}$$

0.29 %

Q1.13



a)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 \cdot \sin 54^\circ = 1.5152 \sin \theta_r$$

$$\sin \theta_r = 0.53393$$

1.13 A ray of light in air is incident at an angle of 54.0° on the smooth surface of a piece of glass. (a) If the refractive index is 1.5152, find the angle of refraction to four significant figures. (b) Find the angle of refraction graphically. (See Fig. P1.13).

Ans. (a) 32.272° , (b) 32.3°

Q1.4

Solution:

$$n = \frac{c}{v} = \frac{3 \times 10^8}{v} = 1.9672 \times 10^8 \text{ m/s}$$

Q1.5

Solution:

$$c = \frac{d}{t} = \frac{1.734 \text{ km}}{1/18000\text{s}} = 3.12 * 10^5 \text{ km /s}$$

$$C = 3 * 10^8 \text{ m/s}$$

$$n = \frac{c}{v}$$

$$1.000234 = \frac{3 * 10^5}{v}$$

$$C = 299931.01586635 = 2.99931015 * 10^5 \text{ km/s}$$

Q1.6

Solution

$$C = d/t$$

$$t = \frac{3.84 * 10^5 \text{ km}}{3 * 10^5 \text{ km/s}} \implies 2t = 1.28 * 2 \text{ sec}$$

$$2t = 2.56 \text{ sec.}$$

Q1.7

Solution:

$$n = \frac{c}{v}$$

$$v = \frac{d}{t} = 1.5 * 10^8 \text{ km}$$

$$t = \frac{d}{v} = \frac{1.5 * 10^8}{3 * 10^5} = 500 \text{ sec.}$$

$$1 \text{ min} = 60 \text{ sec.} \quad = 8 \text{ min } 20 \text{ sec.}$$

Q1.8

Solution:

$$\begin{aligned} \text{Opl} &= \Delta_1 + \Delta_2 + \Delta_3 = n_1 d_1 + n_2 d_2 + n_3 d_3 \\ &= 1.525 * 10 \text{ cm} + 1.333 * 30.5 + 5 * 1.525 = 63.5315 \text{ cm} \end{aligned}$$

1.9

A water tank is 62.0 cm long inside and has glass ends which are each 2.50 cm thick. If the refractive index of water is 1.3330 and of glass is 1.6240, find the overall optical path. **(2-marks)**

$$\begin{aligned} \text{Opl} &= \Delta_1 + \Delta_2 + \Delta_3 = n_1 d_1 + n_2 d_2 + n_3 d_3 \\ &= 2.5 * 1.624 + 62 * 1.333 + 2.5 * 1.624 \\ &= 90.766 \end{aligned}$$

Q1.11

Solution

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 * \sin 10^\circ = 1.5258 \sin \theta_2$$

$$\sin \theta_2 = 0.113807955$$

$$\theta_2 = 6.53487^\circ$$

$$\frac{\phi}{\phi'} = \frac{n'}{n}$$

b)

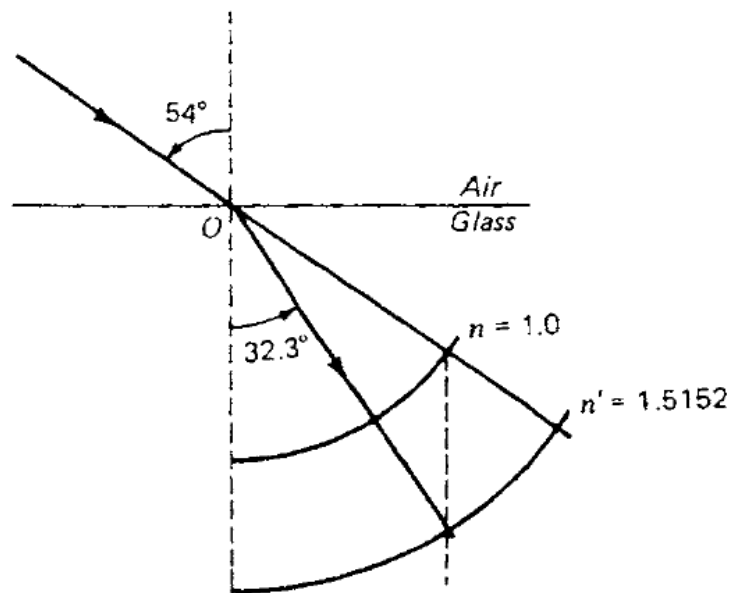
$$\frac{10}{\varphi'} = \frac{1.5258}{1}$$

$$\varphi' = 6.5539^\circ$$

$$\text{ERROE} = 100\% * \frac{6.5539 - 6.5348}{6.5539}$$

0.29 %

Q1.13



a)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 * \sin 54^\circ = 1.5152 \sin \theta_r$$

$$\sin \theta_r = 0.53393$$

1.12 Find the answers to Prob. 1.11, if the angle of incidence is 45.0° and the refractive index is 1.4265.

PROBLEMS

Q1: A ray of light is incident on a piece of glass at an angle of 45.0° . If the angle of refraction is 25.37° , find (a) the refractive index and (b) the critical angle. (c) Solve (b) graphically (See Fig. P2.1). Ans. (a) 1.6504, (b) 37.30° , (c) 1.650 and 37.3°

Solution :

$$\begin{aligned}
 a) \quad n_1 \sin \phi_1 &= n_2 \sin \phi'_r \\
 1 * \sin 45^\circ &= n_2 \sin 25.37^\circ \\
 n_2 &= \frac{\sin 45^\circ}{\sin 25.37^\circ} = \frac{0.707106}{0.42846} = 1.65034
 \end{aligned}$$

$$b) \sin \theta_c = \frac{n_1}{n_2} \sin \theta_i \rightarrow a) \quad n_1 \sin \phi_1 = n_2 \sin \phi'_c$$

$$\begin{aligned}
 \theta_c &= \sin^{-1} \frac{1}{1.6505} \sin 90^\circ \rightarrow = 0.6058 \\
 \theta_c &= 37.29^\circ \approx 37.3^\circ
 \end{aligned}$$

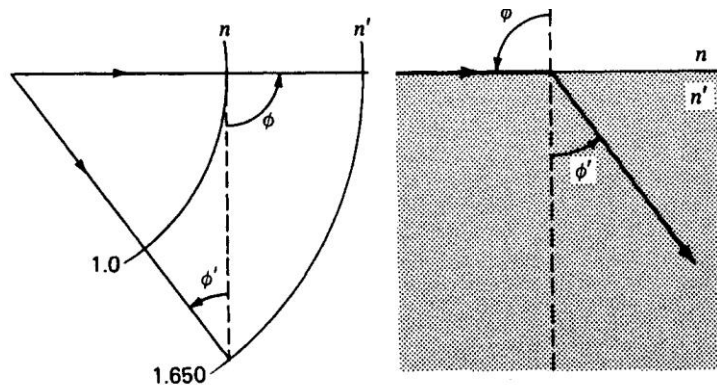


FIGURE P2.1
Graph for Prob. 2.1

Q2: Calculate the lateral displacements of rays of light incident on a block of glass with parallel sides at the following angles: (a) 5.0° , (b) 10.0° , (c) 15.0° , (d) 20.0° , (e) 30.0° , and (f) 40.0° . (g) Plot a graph of versus Φ . Assume the glass thickness to be 5.0 cm

Solution:

a) 5°

$$d = t \sin \theta \left[1 - \frac{n \cos \theta}{n' \cos \theta'} \right]$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta'_r \\ 1 * \sin 5^\circ &= 1.5 \sin \theta'_r \end{aligned}$$

$$\sin \theta' = \frac{1 * \sin 5^\circ}{1.5} = 0.0581$$

$$\theta' = 3.3307^\circ$$

$$\begin{aligned} d &= 5 \sin 5^\circ \left[1 - \frac{1 \cos \theta}{1.5 \cos \theta'} \right] \\ d &= 5 \sin 5^\circ \left[1 - \frac{1 \cos 5^\circ}{1.5 \cos 3.3307^\circ} \right] \\ \therefore d &= 0.1417 \text{ cm} \\ \therefore d &= 1.417 \text{ mm} \end{aligned}$$

b) 10°

$$d = t \sin \theta \left[1 - \frac{n \cos \theta}{n' \cos \theta'} \right]$$

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta'_r \\ 1 * \sin 10^\circ &= 1.5 \sin \theta'_r \end{aligned}$$

$$\sin \phi' = \frac{1 \cdot \sin 5^\circ}{1.5} = 0.0581$$

$$\phi' = 3.3307^\circ$$

$$d = 5 \sin 10^\circ \left[1 - \frac{1}{1.5} \frac{\cos 10^\circ}{\cos 6.65^\circ} \right]$$

$$d = 5 \sin 10^\circ \left[1 - \frac{1}{1.5} \frac{0.9848}{\cos 3.33009933} \right]$$

$$\therefore d = 0.2943 \text{ cm}$$

$$\therefore d = 2.944 \text{ mm}$$

c) 20°

$$d = t \sin \phi \left[1 - \frac{n}{n'} \frac{\cos \phi}{\cos \phi'} \right]$$

$$n_1 \sin \phi_1 = n_2 \sin \phi'_r$$

$$1 * \sin 20^\circ = 1.5 \sin \phi'_r$$

$$\sin \phi' = \frac{1 \cdot \sin 20^\circ}{1.5} = 0.2280$$

$$\phi' = 13.18^\circ$$

$$d = 5 \sin 20^\circ \left[1 - \frac{1}{1.5} \frac{\cos 20^\circ}{\cos 13.18^\circ} \right]$$

$$d = 0.6098$$

$$\therefore d \approx 6.1 \text{ mm}$$

d) 30°

$$d = t \sin \phi \left[1 - \frac{n \cos \phi}{n' \cos \phi'} \right]$$

$$\begin{aligned} n_1 \sin \phi_1 &= n_2 \sin \phi'_r \\ 1 * \sin 30^\circ &= 1.5 \sin \phi'_r \end{aligned}$$

$$\sin \phi' = \frac{1 * \sin 30^\circ}{1.5} = 0.94281$$

$$\phi' = 19.47^\circ$$

$$d = 5 \sin 20^\circ \left[1 - \frac{1 \cos 30^\circ}{1.5 \cos 19.5^\circ} \right]$$

$$d = 0.203 \text{ cm}$$

$$\therefore d \approx 2.03 \text{ mm}$$

e) 40°

$$d = t \sin \phi \left[1 - \frac{n \cos \phi}{n' \cos \phi'} \right]$$

$$\begin{aligned} n_1 \sin \phi_1 &= n_2 \sin \phi'_r \\ 1 * \sin 20^\circ &= 1.5 \sin \phi'_r \end{aligned}$$

$$\sin \phi' = \frac{1 * \sin 20^\circ}{1.5} = 0.42853$$

$$\phi' = 25.374^\circ$$

$$d = 5 \sin 40^\circ \left[1 - \frac{1 \cos 40^\circ}{1.5 \cos 25.374^\circ} \right]$$

b) The angle of deviation $\gamma = \phi_2 - \phi'_2$

$$\phi_1' = \phi'_2$$

$$A = \phi_1' + \phi'_1 \rightarrow 55^\circ - 31.226^\circ = \phi'_2$$

$$\phi'_2 = 23.774^\circ$$

$$n_1 \sin \phi_2 = n_2 \sin \phi'_2$$

$$1 * \sin \phi_2 = 1.6705 \sin 23.774^\circ$$

$$\sin \phi_2 = 0.67342^\circ \rightarrow \phi_2 = 42.3322^\circ$$

$$\gamma = \phi_2 - \phi'_2 = 42.3322^\circ - 23.774^\circ$$

$$\gamma = 18.55$$

$$c) \delta_m = \beta + \gamma = 47.3316^\circ$$

Q4: A 45.0° flint glass prism has a refractive index of 1.6705 for sodium yellow light, and it is adjusted for minimum deviation. Find (a) the angle of minimum deviation and (b) the angle of incidence. (c) Solve graphically.

Solve problem 2.7

$$D_m \text{ or } \delta = A(n - 1)$$

$$=(1.6705-1)*45$$

$$=30.1725$$

$$=30.2^\circ$$

$$b) i = \frac{\delta + A}{2} = \frac{30.2 + 45}{2} = 37.6^\circ$$

Q5: A 60.0° prism produces an angle of minimum deviation of 43.60° for blue light. Find

(a) the refractive index, (b) the angle of refraction, and (c) the angle of incidence.

Solution

$$\frac{n'}{n} = \frac{\sin \frac{1}{2}(\delta_m + \alpha)}{\sin \frac{1}{2}\alpha} = \frac{\delta_m + \alpha}{\alpha}$$

$$\delta = (n' - 1)\alpha$$

Thin prism in air

a)

$$n' = \frac{\sin(43.6 + 60)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 51.8^\circ}{\sin 30^\circ} = 1.5717$$

$$= 1.572$$

b) The angle of refraction

$$A = 2r$$

$$= 60^\circ$$

$$r = A/2 = 60/2 = 30^\circ$$

c) The angle of incidence

$$\delta = 2i - A \quad \text{or} \quad D_m$$

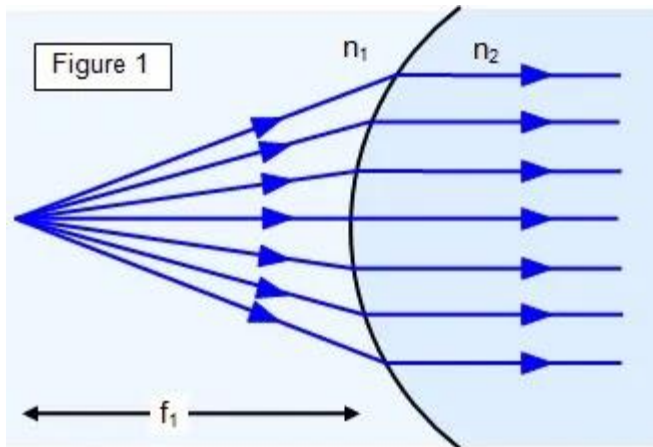
$$2i = \delta + A$$

$$i = \frac{\delta + A}{2} = \frac{43.6 + 60}{2} = 51.8^\circ \text{ the angle of incidence}$$

PROBLEMS

3.1 The left end of a long glass rod of index 1.6350 is ground and polished to a convex spherical surface of radius 2.50 cm. A small object is located in the air and on the axis 9.0 cm from the vertex. Find (a) the primary and secondary focal lengths, (b) the power of the surface, (c) the image distance, and (d) the lateral magnification.

Ans. (a) +3.937 and +6.43cm, (b) +25.40D, (c) +11.44cm, (d) -0.777



a) $\frac{n}{f} = \frac{n_2 - n_1}{R}$

$$\frac{1}{f_1} = \frac{n_2 - 1}{R}$$

$$f_1 = \frac{r}{n_2 - 1}$$

$$f_1 = \frac{2.5}{0.635} = +3.937 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - 1}{R}$$

$$\frac{1.635}{f_2} = \frac{1.635 - 1}{2.5}$$

$$\frac{1.635}{f_2} = \frac{0.635}{2.5}$$

b) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.635 - 1}{0.025} = 25.40 \text{ cm}$

c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

$$\frac{1}{9} + \frac{1.635}{s'} = \frac{1.634 - 1}{2.5}$$

$$\frac{1.635}{s'} = \frac{1.635 - 1}{2.5} - \frac{1}{9}$$

$$S' = +11.443 \text{ cm}$$

(d) The lateral magnification

$$m = -\frac{s' - r}{s + r}$$

$$m = -\frac{11.44 - 2.5}{11.44 + 2.5}$$

$$m = -\frac{8.943}{13.94} = -0.71$$

3.3 The left end of a long plastic rod of index 1.530 is ground and polished to a convex spherical surface of radius 2.650 cm. An object 2.50 cm high is located in the air and on the axis 16.0 cm from the vertex. Find (a) the primary and secondary focal lengths,

(b) the power of the surface, (c) the image distance, and (d) the size of the image,

Solution

a)1-

$$\frac{n_1}{f_1} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{f_1} = \frac{1.53 - 1}{2.65}$$

$$\frac{1}{f_1} = \frac{0.53}{2.65}$$

$$f_1 = +5 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - 1}{R}$$

$$\frac{1.53}{f_2} = \frac{1.53 - 1}{2.65} = \frac{0.53}{2.65}$$

$$f_2 = +7.65 \text{ cm}$$

b) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.53 - 1}{0.0265}$

$$p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.53 - 1}{0.0265} = \frac{0.53}{0.0265}$$

$$P = +20 \text{ Diopter}$$

c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f}$$

$$\frac{1}{16} + \frac{1.53}{s'} = \frac{1.53 - 1}{2.65}$$

$$\frac{1.53}{s'} = \frac{0.53}{2.65} - \frac{1}{16}$$

$$S' = +11.12 \text{ cm}$$

(d) the size of the image,

The lateral magnification

$$m = -\frac{s' - r}{s + r} = \frac{y'}{y}$$

$$-\frac{11.12 - 2.65}{16 + 2.65} = \frac{y'}{y}$$

$$-\frac{8.4772}{18.65} = \frac{y'}{2.5}$$

$$Y' = -1.136 \text{ cm}$$

3.5 The left end of a water trough has a transparent surface of radius - 2.0 cm. A small object 2.5 cm high is located in the air and on the axis 10.0 cm from the vertex. Find (a) the primary and secondary focal lengths, (b) the power of the surface, (c) the imagedistance, and (d) the size of the image. Assume water to have an index 1.3330.

Ans. (a) - 6.01 and - 8.01 em, (b) -16.65 0, (c) - 5.0 cm, (d) +0.938 cm

a) **solution**

a)1-

$$\frac{1}{f_1} = \frac{n_2 - 1}{R}$$

$$\frac{1}{f_1} = \frac{1.33 - 1}{-2}$$

$$\frac{1}{f_1} = \frac{0.33}{-2}$$

$$f_1 = -6.01 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - n_1}{R}$$

$$\frac{1.33}{f_2} = \frac{1.33 - 1}{-2} = \frac{0.33}{-2}$$

$$f_2 = -8.01 \text{ cm}$$

b) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.333 - 1}{-0.02}$

OR $p = \frac{1}{f(m)} = -\frac{1}{6.01} = -16.65 \text{ D}$

c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f}$$

$$b) \frac{1}{10} + \frac{1.33}{s'} = \frac{1.333-1}{-2}$$

$$\frac{1.33}{s'} = \frac{0.33}{-2} - \frac{1}{10}$$

$$c) S' = - 5.001 \text{ cm}$$

d)

(d) the size of the image,

e) The lateral magnification

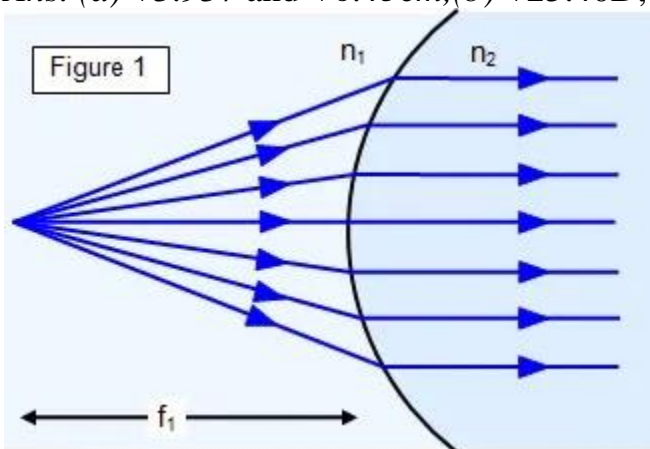
$$f) m = -\frac{s'-r}{s+r} = \frac{y'}{y}$$

$$= -\frac{-5+2}{10-2} = \frac{y'}{2.5}$$

PROBLEMS

3.1 The left end of a long glass rod of index 1.6350 is ground and polished to a convex spherical surface of radius 2.50 cm. A small object is located in the air and on the axis 9.0 cm from the vertex. Find (a) the primary and secondary focal lengths, (b) the power of the surface, (c) the image distance, and (d) the lateral magnification.

Ans. (a) +3.937 and +6.43cm, (b) +25.40D, (c) +11.44cm, (d) -0.777



$$d) \frac{n}{f} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{f_1} = \frac{n_2 - 1}{R}$$

$$f_1 = \frac{r}{n_2 - 1}$$

$$f_1 = \frac{2.5}{0.635} = +3.937 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - 1}{R}$$

$$\frac{1.635}{f_2} = \frac{1.635 - 1}{2.5}$$

$$\frac{1.635}{f_2} = \frac{0.635}{2.5}$$

e) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.635 - 1}{0.025} = 25.40 \text{ cm}$

f) c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

$$\frac{1}{9} + \frac{1.635}{s'} = \frac{1.635 - 1}{2.5}$$

$$\frac{1.635}{s'} = \frac{1.635 - 1}{2.5} - \frac{1}{9}$$

$$S' = +11.443 \text{ cm}$$

(d) The lateral magnification

$$m = -\frac{s' - r}{s + r}$$

$$m = -\frac{11.44 - 2.5}{11.44 + 2.5}$$

$$m = -\frac{8.943}{13.94} = -0.71$$

3.3 The left end of a long plastic rod of index 1.530 is ground and polished to a convex spherical surface of radius 2.650 cm. An object 2.50 cm high is located in the air and on the axis 16.0 cm from the vertex. Find (a) the primary and secondary focal lengths, (b) the power of the surface, (c) the image distance, and (d) the size of the image,

Solution

a)1-

$$\frac{n_1}{f_1} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{f_1} = \frac{1.53 - 1}{2.65}$$

$$\frac{1}{f_1} = \frac{0.53}{2.65}$$

$$f_1 = +5 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - 1}{R}$$

$$\frac{1.53}{f_2} = \frac{1.53 - 1}{2.65} = \frac{0.53}{2.65}$$

$$f_2 = +7.65 \text{ cm}$$

b) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.53 - 1}{0.0265}$

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$$P = +20 \text{ Diopter}$$

c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f}$$

$$\frac{1}{16} + \frac{1.53}{s'} = \frac{1.53-1}{2.65}$$

$$\frac{1.53}{s'} = \frac{0.53}{2.65} - \frac{1}{16}$$

$$S' = +11.12 \text{ cm}$$

(d) the size of the image,

The lateral magnification

$$m = -\frac{s'-r}{s+r} = \frac{y'}{y}$$

$$-\frac{11.12 - 2.65}{16 + 2.65} = \frac{y'}{y}$$

$$-\frac{8.4772}{18.65} = \frac{y'}{2.5}$$

$$Y' = -1.136 \text{ cm}$$

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Ans. (a) - 6.01 and - 8.01 em, (b) -16.65 0, (c) - 5.0 cm, (d) +0.938 cm

g) solution

a)1-

$$\frac{1}{f_1} = \frac{n_2 - 1}{R}$$

$$\frac{1}{f_1} = \frac{1.33 - 1}{-2}$$

$$\frac{1}{f_1} = \frac{0.33}{-2}$$

$$f_1 = -6.01 \text{ cm}$$

2) the secondary focal length

$$\frac{n_2}{f_2} = \frac{n_2 - n_1}{R}$$

$$\frac{1.33}{f_2} = \frac{1.33 - 1}{-2} = \frac{0.33}{-2}$$

$$f_2 = -8.01 \text{ cm}$$

b) the power of the surface $p = \frac{1}{f(m)} = \frac{n_2 - 1}{R} = \frac{1.333 - 1}{-0.02}$

OR $p = \frac{1}{f(m)} = -\frac{1}{6.01} = -16.65 \text{ D}$

c) the image distance

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f}$$

h) $\frac{1}{10} + \frac{1.33}{s'} = \frac{1.333 - 1}{-2}$

$$\frac{1.33}{s'} = \frac{0.33}{-2} - \frac{1}{10}$$

i) $S' = -5.001 \text{ cm}$

j)

(d) the size of the image,

k) The lateral magnification

l) $m = -\frac{s' - r}{s + r} = \frac{y'}{y}$
 $-\frac{-5 + 2}{10 - 2} = \frac{y'}{2.5}$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$+0.375 = \frac{y'}{2.5}$$

$$\mathbf{Y = +0.9375 \text{ cm}}$$

$$\mathbf{Y' = +.938 \text{ cm}}$$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$-\frac{-3}{8} = \frac{y'}{2.5}$$

$$+0.375 = \frac{y'}{2.5}$$

$$\mathbf{Y = +0.9375 \text{ cm}}$$

$$\mathbf{Y' = +.938 \text{ cm}}$$

4.1 An object located 12.0 cm in front of a thin lens has its image formed on the opposite side 42.0 cm from the lens. Calculate (a) the focal length of the lens and (b) the lens power. Ans. (a) +9.33 cm, (b) + 10.72 D

Solution

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

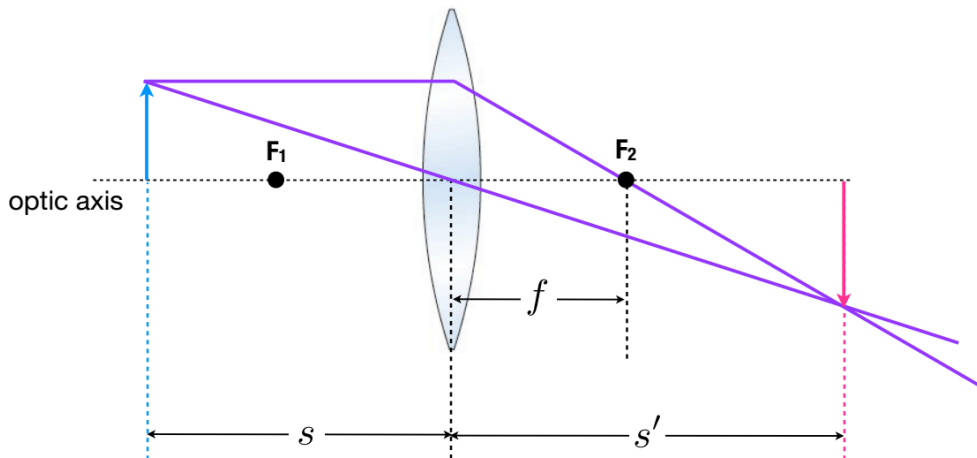
$$a) f = \frac{uv}{u+v} = \frac{42 \cdot 12}{42+12} = +\frac{504}{54} = +9.333 \text{ cm}$$

$$b) P = \frac{1}{f(m)} = \frac{100}{9.333} = 10.72 \text{ D}$$

4.2 An object 2.50 cm high is placed 12.0 cm in front of a thin lens of focal length 3.0 cm.

Calculate (a) the image distance, (b) the magnification, and (c) the nature of the image. (d) Check your answers by a graph.

→ we draw the rays as though they refract at the center - this is OK for 'thin' lenses



Solution

a)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$a) v = \frac{fu}{u-f} = \frac{3 \cdot 12}{12-3} = +\frac{36}{9} = +4 \text{ cm}$$

$$b) P = \frac{1}{f(m)} = \frac{100}{3} = 33.33 \text{ D}$$

$$c) m = -\frac{v}{u} = -\frac{4}{12} = -\frac{1}{3}$$

We know this lens is real, inverted and minimized

4.3 The two faces of a thin lens have radii $r_1 = + 10.0$ cm and $r_2 = - 25.0$ cm, respectively. The lens is made of glass of index 1.740. Calculate (a) the focal length and (b) the power of the lens.

Solution

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\frac{1}{f} = (1.74 - 1) \left(\frac{1}{10} - \frac{1}{-25} \right)$$

$$\frac{1}{f} = (1.74 - 1) \left(\frac{1}{10} + \frac{1}{25} \right)$$

$$\frac{1}{f} = (0.74) \left(\frac{1}{10} + \frac{1}{25} \right)$$

$$\frac{1}{f} = (0.74) \left(\frac{25+10}{250} \right) = \frac{25.9}{250}$$

$$\frac{1}{f} = (0.74) \left(\frac{35}{250} \right)$$

$$f = 9.65 \text{ cm}$$

$$\text{b) } P = \frac{1}{f(m)} = \frac{100}{9.653} = 10.36 \text{ D}$$

4.4 An object 3.50 cm high is located 10.0 cm in front of a lens whose focal length $f = - 6.0$ cm. Calculate (a) the power of the lens, (b) the image distance, and (c) the lateral magnification. Graphically locate the image by (d) the parallel-ray method and (e) the oblique-ray method.

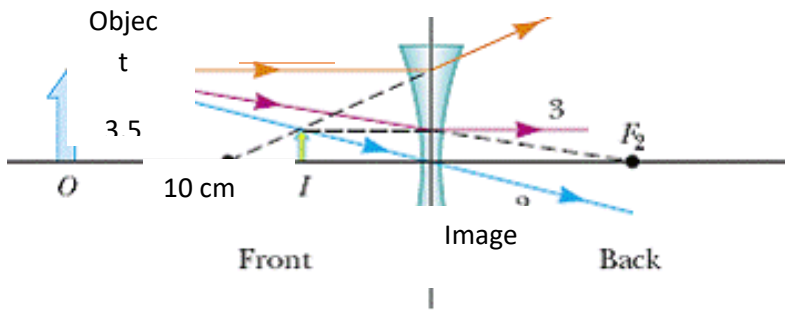
Solution

$$a) P = \frac{1}{f(m)} = -\frac{100}{6} = -16.66 \text{ D}$$

$$b) v' = \frac{uxf}{u+f} = \frac{10x-6}{10+6} = -\frac{60}{16} = -3.75 \text{ cm}$$

$$c) m = -\frac{v}{u} = \frac{3.75}{10} = 0.375$$

d)



d)

4.5 An equiconcave lens is to be made of flint glass of index 1.750. Calculate the radii of curvature if it is to have a power of - 3.0 D. *Ans.* Both 50.0-cm radius

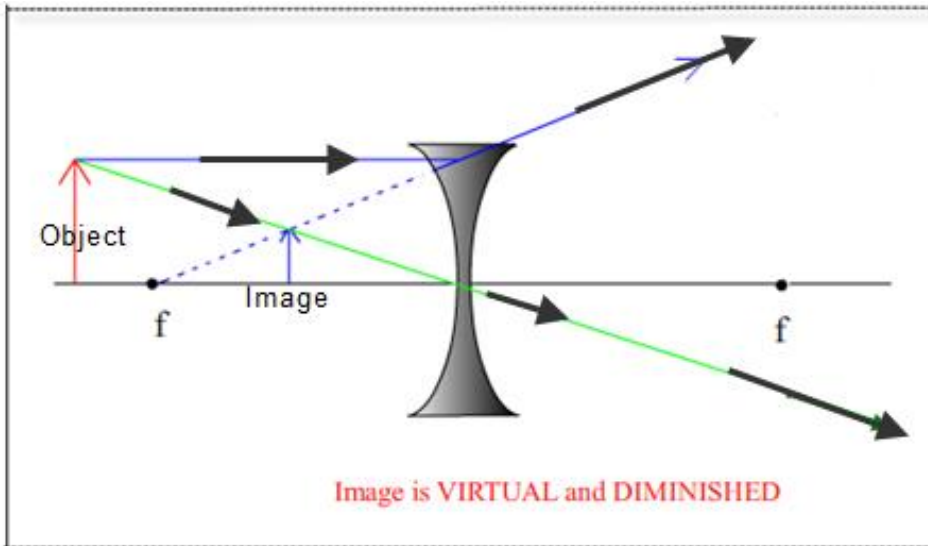
Solution

$$P = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$-3 = (1.75 - 1) \left(\frac{-1}{r} - \frac{-1}{r} \right)$$

$$-3 = \frac{-0.75 \times 2}{r}$$

$$r = 0.5 \text{ m} \quad \longrightarrow \quad r = 50 \text{ cm}$$



4.6 A plano-convex lens is to be made of light flint glass of index 1.680. Calculate the radius of curvature necessary to give the lens a power of 4.5 D

$$P = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\boxed{=0}$$

$$p = (n - 1) \left(\frac{1}{r_1} - \frac{1}{\infty} \right)$$

$$p = \frac{1.68 - 1}{r_1}$$

$$4.5 = \frac{0.68}{r_1}$$

$r_1 = 15.11 \text{ cm}$ the radius of curvature

4.7 Two lenses with focal lengths $f_1 = +5.0 \text{ cm}$ and $f_2 = +10.0 \text{ cm}$ are located 5.0 cm apart. If an object 2.50 cm high is located 15.0 cm in front of the first lens, find (a) the position and (b) the size of the final image.

Ans. (a) +2.00 cm from second lens, (b) -1.0 cm

Solution

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1}$$

$$v_{11} = \frac{f_1 x u_1}{u_1 - f_1} = \frac{15 \times 5}{15 - 5} = +\frac{75}{10} = +7.5 \text{ cm}$$

$$u_{o2} = d - v_1 = 5 - 7.5 = -2.5 \text{ cm}$$

$$v_2 = \frac{f_2 x u_2}{u_2 - f_2} = \frac{10 \times -2.5}{-2.5 - 10} = \frac{+25}{-12.5} = +2$$

$$m_t = m_1 m_2 = \left(\frac{-v_1}{u_1} \right) \left(\frac{-v_2}{u_2} \right) = \left(\frac{-7.5}{15} \right) \left(\frac{+2}{-2.5} \right)$$

$$\frac{y'}{y} = \frac{-15}{15 \times 2.5}$$

$$\frac{y'}{2.5} = \frac{-1}{2.5}$$

$$y' = -1 \text{ cm}$$

4.8 A converging lens is used to focus a sharp image of a candle flame on a screen. Without moving the candle flame a second lens with radii $r_1 = +10.0$ cm and $r_2 = -20.0$ cm and index 1.650 is placed in the converging beam 30.0 cm from the screen. (a) Calculate the power of the second lens. (b) How close to the second lens should the screen now be placed to obtain a sharp image of the flame? (c) Make a graph of this experiment

4. 4.8 A converging lens is used to focus a sharp image of a candle flame on a screen. Without moving the candle flame a second lens with radii $r_1 = +10.0$ cm and $r_2 = -20.0$ cm and index 1.650 is placed in the converging beam 30.0 cm from the screen. (a) Calculate the power of the second lens. (b) How close to the second lens should the screen now be placed to obtain a sharp image of the flame? (c) Make a graph of this experiment.

4.9 A double-convex lens is to be made of glass having a refractive index of 1.580. If one surface is to have twice the radius of the other and the focal length is to be $+6.0$ cm, find the radii.

Solution

$$\begin{aligned} 1. \quad R_1 &= 2R_2 & p &= 1/f(m) & \therefore f &= 6 \text{ cm} \\ 2. \quad \frac{1}{f} &= (n' - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\ 3. \quad \frac{1}{6} &= (1.58 - 1) \left(\frac{1}{2R_2} + \frac{1}{R_2} \right) \end{aligned}$$

$$R_2 = 5.22 \text{ cm}, R_1 = 10.44 \text{ cm}$$

$$R_1 = 2R_2 = 2 * 3.9$$

$$R_1 = 7.8 \text{ cm}$$

4.10 Two lenses having focal lengths $f_1 = +9.0 \text{ cm}$ and $f_2 = -18.0 \text{ cm}$ are placed 3.0 cm apart. If an object 2.50 cm high is located 20.0 cm in front of the first lens, calculate (a) the position and (b) the size of the final image.

Solve problems

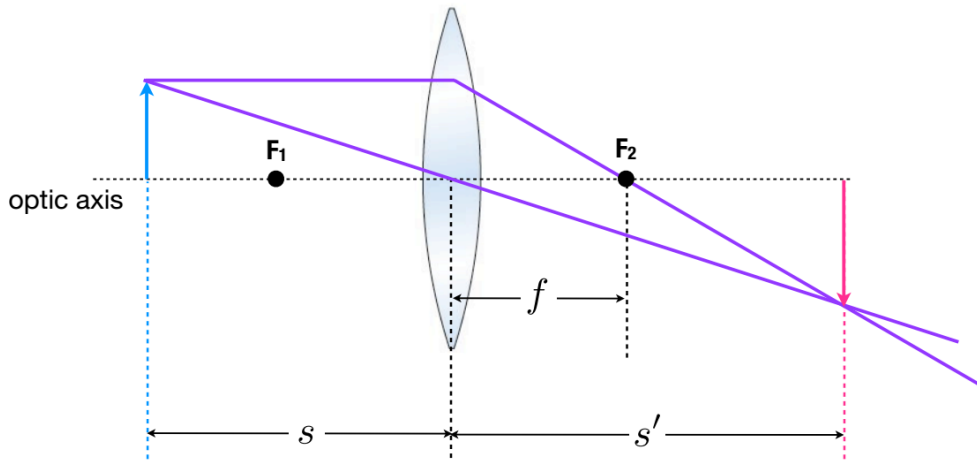
Q1: An object located 12.0 cm in front of a thin lens has its image formed on the opposite side 42.0 cm from the lens. Calculate (a) the focal length of the lens and (b) the lens power.

Solution

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
$$c) \quad f = \frac{uv}{u+v} = \frac{42 * 12}{42+12} = +\frac{504}{54} = +9.333 \text{ cm}$$
$$d) \quad P = \frac{1}{f(m)} = \frac{100}{9.333} = 10.72 \text{ D}$$

Q2: An object 2.50 cm high is placed 12.0 cm in front of a thin lens of focal length 3.0 cm . Calculate (a) the image distance, (b) the magnification, and (c) the nature of the image. (d) Check your answers by a graph.

→ we draw the rays as though they refract at the center - this is OK for 'thin' lenses



Solution

a)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$d) v = \frac{fxu}{u-f} = \frac{3 \cdot 12}{12-3} = +\frac{36}{9} = +4 \text{ cm}$$

$$e) P = \frac{1}{f(m)} = \frac{100}{3} = 33.33 \text{ D}$$

$$f) m = -\frac{v}{u} = -\frac{4}{12} = -\frac{1}{3}$$

We know this lens is real, inverted and minimized

Q3: The two faces of a thin lens have radii $r_1 = +10.0 \text{ cm}$ and $r_2 = -25.0 \text{ cm}$, respectively. The lens is made of glass of index 1.740. Calculate (a) the focal length and (b) the power of the lens.

Solution

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\frac{1}{f} = (1.74 - 1) \left(\frac{1}{10} - \frac{1}{-25} \right)$$

$$\frac{1}{f} = (1.74 - 1) \left(\frac{1}{10} - \frac{1}{-25} \right)$$

$$\frac{1}{f} = (0.74) \left(\frac{1}{10} + \frac{1}{25} \right)$$

$$\frac{1}{f} = (0.74) \left(\frac{25+10}{250} \right) = \frac{25.9}{250}$$

$$\frac{1}{f} = (0.74) \left(\frac{35}{250} \right)$$

$$f = 9.65 \text{ cm}$$

$$c) P = \frac{1}{f(m)} = \frac{100}{9.653} = 10.36 \text{ D}$$

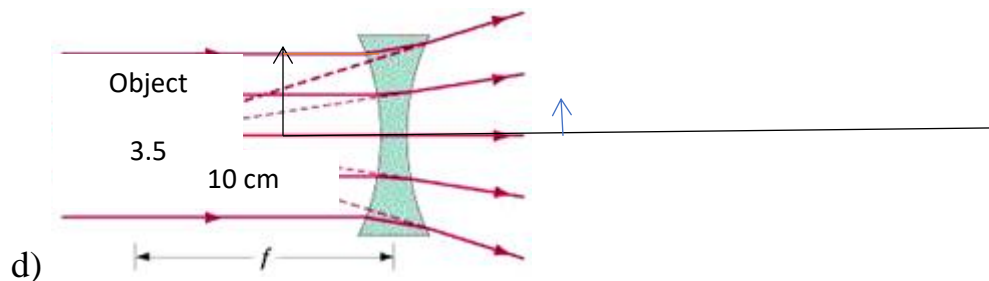
Q4: An object 3.50 cm high is located 10.0 cm in front of a lens whose focal length $f = -6.0$ cm. Calculate (a) the power of the lens, (b) the image distance, and (c) the lateral magnification. Graphically locate the image by (d) the parallel-ray method and (e) the oblique-ray method.

Solution

$$b) P = \frac{1}{f(m)} = - \frac{100}{6} = -16.66 \text{ D}$$

$$b) v' = \frac{uf}{u+f} = \frac{10 \times -6}{10-6} = -\frac{60}{4} = -15 \text{ cm}$$

$$c) m = -\frac{v}{u} = \frac{15}{10} = 1.5$$



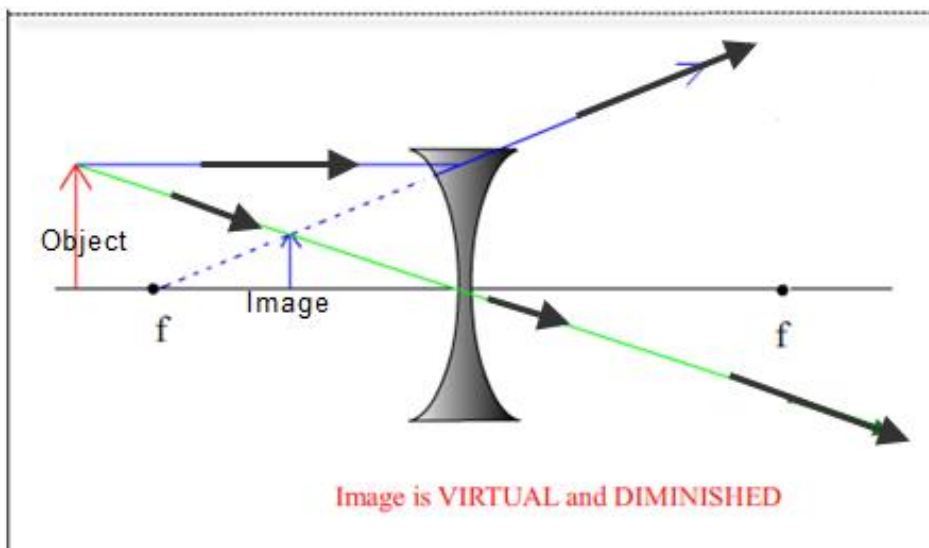
Q5: An equiconcave lens is to be made of flint glass of index 1.750. Calculate the radii of curvature if it is to have a power of - 3.0 D.

Solution

$$P = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$-3 = (1.75 - 1) \left(\frac{-1}{r} - \frac{-1}{r} \right)$$

$$-3 = \frac{-0.75 \times 2}{r} \qquad r = 0.5 \text{ m} \quad \longrightarrow \quad r = 50 \text{ cm}$$



Q6: A plano-convex lens is to be made of light flint glass of index 1.680. Calculate the radius of curvature necessary to give the lens a power of 4.5 D

$$P = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$p = (n - 1) \left(\frac{1}{r_1} - \frac{1}{\infty} \right)$$

$$p = \frac{1.68 - 1}{r_1}$$

$$4.5 = \frac{0.68}{r_1}$$

$r_1 = 15.11 \text{ cm}$ the radius of curvature

Q7: Two lenses with focal lengths $f_1 = +5.0 \text{ cm}$ and $f_2 = +10.0 \text{ cm}$ are located 5.0 cm apart. If an object 2.50 cm high is located 15.0 cm in front of the first lens, find (a) the position and (b) the size of the final image.

Solution

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1}$$

$$v_{11} = \frac{f_1 x u_1}{u_1 - f_1} = \frac{15 x 5}{15 - 5} = +\frac{75}{10} = +7.5 \text{ cm}$$

$$u_{o2} = d - v_1 = 5 - 7.5 = -2.5 \text{ cm}$$

$$v_2 = \frac{f_2 x u_2}{u_2 - f_2} = \frac{10 x -2.5}{-2.5 - 10} = \frac{+25}{-12.5} = +2$$

$$m_t = m_1 m_2 = \left(\frac{-v_1}{u_1}\right) \left(\frac{-v_2}{u_2}\right) = \left(\frac{-7.5}{15}\right) \left(\frac{+2}{-2.5}\right)$$

$$\frac{y'}{y} = \frac{-15}{15 x 2.5}$$

$$\frac{y'}{2.5} = \frac{-1}{2.5}$$

$$y' = -1 \text{ cm}$$

Q8: A converging lens is used to focus a sharp image of a candle flame on a screen. Without moving the candle flame a second lens with radii $r_1 = +10.0 \text{ cm}$ and $r_2 = -20.0 \text{ cm}$ and index 1.650 is placed in the converging beam 30.0 cm from the screen. (a) Calculate the power of the second lens. (b) How close to the second lens should the screen now be placed to obtain a sharp image of the flame? (c) Make a graph of this experiment

4.10 Two lenses having focal lengths $f_1 = +9.0$ cm and $f_2 = -18.0$ cm are placed 3.0 cm apart. If an object 2.50 cm high is located 20.0 cm in front of the first lens, calculate (a) the position and (b) the size of the final image. (c) Check your answer graphically.

Solution:

$$\frac{1}{s_{o1}} + \frac{1}{s'_{i1}} = \frac{1}{f_1}$$

$$a) s'_{i1} = \frac{f_1 x s_{o1}}{s_{o1} - f_1} = \frac{9 \times 20}{20 - 9} = + \frac{180}{11} = +16.363 \text{ cm}$$

$$s_{o2} = 3 - 16 = 3 - 16.363 = -13.363 \text{ cm}$$

The position of the final image

$$s'_{i2} = \frac{f_2 x s_{o2}}{s_{o2} - f_2} = \frac{-18 \times -13.636}{-13.636 + 18} = \frac{+240.545}{4.637} = +51.875 \text{ cm}$$

$$m_t = m_1 m_2 = \left(\frac{-s'_{i1}}{s_{o1}} \right) \left(\frac{-s'_{i2}}{s_{o2}} \right) = \left(\frac{-16.363}{20} \right) \left(\frac{-51.875}{-13.636} \right)$$

(b) The size of the final image

$$\frac{y'}{y} = -3.1124$$

$$\frac{y'}{2.5} = -3.1124$$

$$y' = -7.781 \text{ cm}$$

4.11 A lantern slide 8.0 cm high is located 3.50 m from a projection screen. What is the focal length of the lens that will be required to project an image 1.0 m high?

4.12 An object is located 1.60 m from a white screen. A lens of what focal length will be required to form a real and inverted image on the screen with a magnification of - 6.0?

Ans. 19.59 cm

Thin lens equation is $\frac{1}{u_o} + \frac{1}{v_i} = \frac{1}{f}$

$$\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f}$$

$$\frac{1}{x} + \frac{1}{1.6-x} = \frac{1}{f}$$

The lateral magnification $m = -\frac{v_i}{u_o}$

$$-6 = -\frac{1.6-x}{x}$$

The distance of object $u_o = x = 0.2286$ m

The distance of image $v_i = 1.6 - x = 1.3714$ m

$$\frac{D}{x(D-x)} = \frac{1}{f}$$

$$\frac{1.6}{0.2286(1.3714)} = \frac{1}{f}$$

$$\therefore f = 0.1959 \text{ m} = 19.59 \text{ cm}$$

4.13 Three thin lenses have powers + 1.50, - 2.80, and 3.40 D, respectively. What are all the possible powers that can be obtained with these three lenses using one, two, or three at a time in contact?

4.14 Two thin lenses having the following radii of curvature and index are placed in contact. For the first lens $r_1 = +12.0$ cm $r_2 = -18.0$ cm, and $n = 1.560$, and for the second lens $r_1 = -30.0$ cm, $r_2 = +20.0$ cm, and $n = 1.650$. Find their (a) individual powers, (b) combined power, (c) individual focal lengths, and (d) combined focal length.

Solution:

$$P = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

a)

$$1 - P_1 = (1.56 - 1) \left(\frac{1}{0.12} - \left(-\frac{1}{0.18} \right) \right)$$

$$P_1 = 7.78 \text{ D}$$

$$2 - P_2 = (1.65 - 1) \left(\frac{1}{0.3} - \left(\frac{1}{0.2} \right) \right)$$

$$P_2 = -5.41 \text{ D}$$

$$P = P_1 + P_2$$

$$P = +2.361 \text{ Diopter}$$

4.15 An object 2.50 cm high is located 15.0 cm in front of a lens of +5.0 cm focal length. A lens with a focal length of -12.0 cm is placed 2.50 cm beyond this converging lens. Find (a) the position and (b) the size of the final image.

Ana. (a) +8.57 cm, (b) -2.143 cm

4.16 An object 2.50 cm high is located 8.0 cm in front of a lens of -2.40 cm focal length. A lens of +5.0 cm focal length is placed 1.50 cm behind the first lens. Find (a) the position and (b) the size of the final image

$$\frac{1}{s_{o1}} + \frac{1}{s'_{i1}} = \frac{1}{f_1}$$

$$s'_{i1} = \frac{f_1 s_{o1}}{s_{o1} - f_1} = \frac{9 \times 20}{20 - 9} = +\frac{180}{11} = +16.363 \text{ cm}$$

$$s_{o2} = a - s'_{i1} = 5 - 7.5 = -2.5 \text{ cm}$$

The position of the final image

$$s'_{i2} = \frac{f_2 x s_{o2}}{s_{o2} - f_2} = \frac{-18 \times -13.636}{-13.636 + 18} = \frac{+240.545}{4.637} = +51.875 \text{ cm}$$

$$m_t = m_1 m_2 = \left(\frac{-s'_{i1}}{s_{o1}} \right) \left(\frac{-s'_{i2}}{s_{o2}} \right) = \left(\frac{-16.363}{20} \right) \left(\frac{-51.875}{-13.636} \right)$$

(b) The size of the final image

$$\frac{y'}{y} = -3.1124$$

$$\frac{y'}{2.5} = -3.1124$$

$$y' = -7.781 \text{ cm}$$

4.17 Three lenses with focal lengths of +8.40, -4.60, and +6.20 cm, respectively, are located one behind each other in this order and 2.0 cm apart. (a) If parallel light is incident on the first lens, how far behind the third lens will the light be brought to a focus? (b) Draw a scale diagram.

4.18 An object 3.50 cm high is located 8.0 cm in front of a lens of -7.0 cm focal length. A lens of +4.50 cm focal length is placed 3.5 cm behind the first lens. Find (a) the position and (b) the size of the image. (c) Make a diagram to scale

$$\frac{1}{s_{o1}} + \frac{1}{s'_{i1}} = \frac{1}{f_1}$$

$$s'_{i1} = \frac{f_1 x s_{o1}}{s_{o1} - f_1} = \frac{-7 \times 8}{8 + 7} = + \frac{180}{11} = -3.733 \text{ cm}$$

$$s_{o2} = a - s'_{i1} = 3.5 + 3.733 = 7.233 \text{ cm}$$

The position of the final image

$$s'_{i2} = \frac{f_2 x s_{o2}}{s_{o2} - f_2} = \frac{4.5 \times 7.233}{7.233 - 4.5} = \frac{+32.5485}{2.733} = +11.91 \text{ cm}$$

$$m_t = m_1 m_2 = \left(\frac{-s'_{i1}}{s_{o1}} \right) \left(\frac{-s'_{i2}}{s_{o2}} \right) = \left(\frac{-11.91}{7.233} \right) \left(\frac{3.733}{8} \right)$$

(b) The size of the final image

$$\frac{y'}{y} = -0.7683$$

$$\frac{y'}{3.5} = -0.7683$$

$$y' = -2.68 \text{ cm}$$

4.7 Two lenses with focal lengths $f_1 = +5.0$ cm and $f_2 = +10.0$ cm are located 5.0 cm apart. If an object 2.50 cm high is located 15.0 cm in front of the first lens, find (a) the position and (b) the size of the final image.

Ans. (a) +2.00 cm from second lens, (b) -1.0 cm

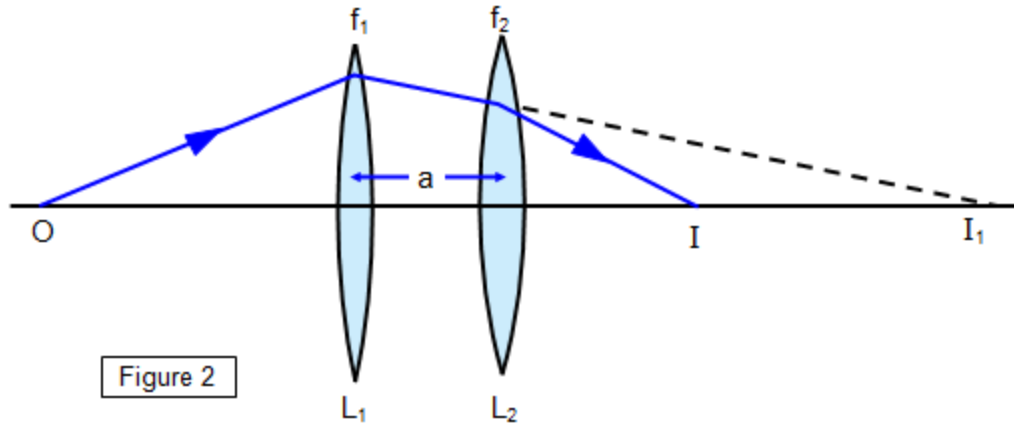


Figure 2

Solution

$$\frac{1}{s_{o1}} + \frac{1}{s'_{i1}} = \frac{1}{f_1}$$

$$s'_{i1} = \frac{f_1 x s_{o1}}{s_{o1} - f_1} = \frac{15 \times 5}{15 - 5} = +\frac{75}{10} = +7.5 \text{ cm}$$

$$s_{o2} = a - s'_{i1} = 5 - 7.5 = -2.5 \text{ cm}$$

The position of the final image

$$s'_{i2} = \frac{f_2 x s_{o2}}{s_{o2} - f_2} = \frac{10 \times -2.5}{-2.5 - 10} = \frac{+25}{-12.5} = +2 \text{ cm}$$

$$m_t = m_1 m_2 = \left(\frac{-s'_{i1}}{s_{o1}} \right) \left(\frac{-s'_{i2}}{s_{o2}} \right) = \left(\frac{-7.5}{15} \right) \left(\frac{+2}{-2.5} \right)$$

(b) The size of the final image

$$\frac{y'}{y} = \frac{-15}{15 \times 2.5}$$

$$\frac{y'}{2.5} = \frac{-1}{2.5}$$

$$y' = -1 \text{ cm}$$

Q 4.9 A double-convex lens is to be made of glass having a refractive index of 1.580. If one surface is to have twice the radius of the other and the focal length is to be +6.0 cm, find the radii.

Solution

$$4. R_1 = 2R_2 \quad p = 1/f(m) \quad \therefore f = 6 \text{ cm}$$

$$5. \frac{1}{f} = (n' - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$6. \frac{1}{6} = (1.58 - 1) \left(\frac{1}{2R_2} + \frac{1}{R_2} \right)$$

$$R_2 = 5.22 \text{ cm}, R_1 = 10.44 \text{ cm}$$

$$R_1 = 2R_2 \quad = 2 * 5.22$$

$$R_1 = 10.44 \text{ cm}$$

4.11 A lantern slide 8.0 cm high is located 3.50 m from a projection screen. What is the focal length of the lens that will be required to project an image 1.0 m high?

Solution

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s} = \frac{y'}{y}$$

$$-\frac{s'}{3.5 \text{ m}} = \frac{1 \text{ m}}{0.08 \text{ m}}$$

$$S' = - 43.74$$

$$f = \frac{s'xs}{s - s'}$$

$$f = \frac{-43.75 \times 0.08 \text{ m} * \text{m}}{0.08 - (-43.75)\text{m}}$$

$$f = \frac{3.5 \text{ m}}{43.83}$$

$$f = 0.079 \text{ m}$$

$$f = 7.98 \text{ cm}$$

4.13 Three thin lenses have powers + 1.50, - 2.80, and 3.40 D, respectively. What are all the possible powers that can be obtained with these three lenses using one, two, or three at a time in contact?

Solution

$$P = p_1 + p_2 + p_3$$

$$= +1.5 - 2.8 + 3.4$$

$$P = 2.1 \text{ D}$$

4.15 An object 2.50 cm high is located 15.0 cm in front of a lens of + 5.0 cm focal length. A lens with a focal length of -12.0 cm is placed 2.50 cm beyond this converging lens. Find (a) the position and (b) the size of the final image.

Ans. (a) +8.57 cm, (b) -2.143 cm

Solution

$$\frac{1}{s_{o1}} + \frac{1}{s'_{i1}} = \frac{1}{f_1}$$

$$s'_{i1} = \frac{f_1 \times s_{o1}}{s_{o1} - f_1} = \frac{15 \times 5}{15 - 5} = +\frac{75}{10} = +7.5 \text{ cm}$$

$$s_{o2} = a - s'_{i1} = 2.5 - 7.5 = -5 \text{ cm}$$

The position of the final image

$$s'_{i2} = \frac{f_2 \times s_{o2}}{s_{o2} - f_2} = \frac{-12 \times -5}{-5 + 12} = \frac{+60}{7} = +8.57 \text{ cm}$$

$$m_t = m_1 m_2 = \left(\frac{-s'_{i1}}{s_{o1}} \right) \left(\frac{-s'_{i2}}{s_{o2}} \right) = \left(\frac{-7.5}{15} \right) \left(\frac{-8.57}{-5} \right)$$

(b) The size of the final image

$$\frac{y'}{2.5} = \frac{-64.275}{7.5}$$

$$y' = -2.1425 \text{ cm}$$

The properties of the final image is **inverted real and minimized**

4.17 Three lenses with focal lengths of +8.40, -4.60, and +6.20 cm, respectively, are located one behind each other in this order and 2.0 cm apart. (a) If parallel light is incident on the first lens, how far behind the third lens will the light be brought to a focus? (b) Draw a scale diagram

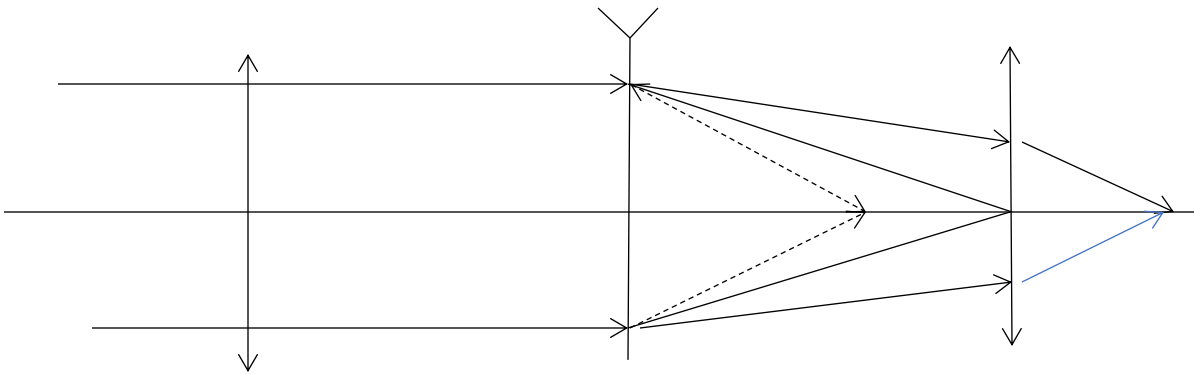
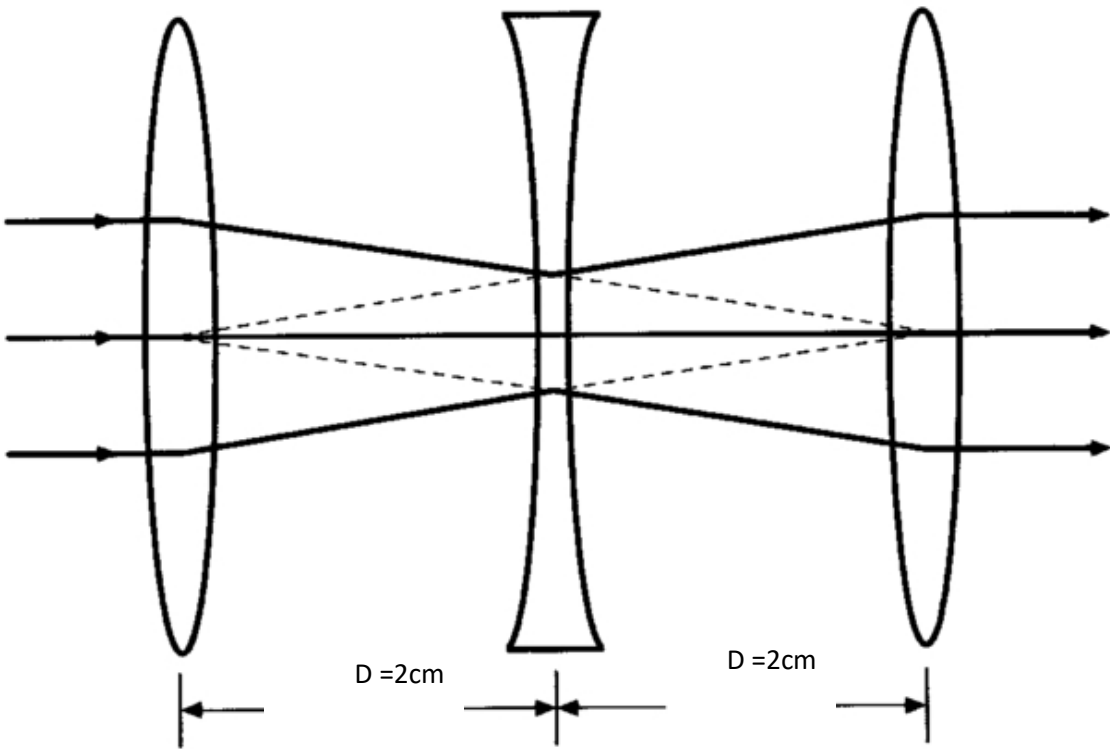
Solution:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$
$$\frac{1}{f} = \frac{1}{8.4} + \frac{1}{-4.6} - \frac{2}{8.4 * -4.6}$$

$$F_{\text{system}} = -21.5 \text{ cm}$$

$$\frac{1}{f_{\text{final}}} = \frac{1}{-21.5} + \frac{1}{6.2} - \frac{2}{-21.5 * 6.2}$$

$$f_{\text{final optical system}} = + 7.7 \text{ cm}$$



PROBLEMS

5.1 An equiconvex lens located in air has radii of 5.20 cm, an index of 1.680, and a thickness of 3.50 cm. Calculate (a) the focal length and (b)

the power of the lens. Find (c) the distances from the vertices to the focal points and (d) the principal points.

Ans. (a) +4.43 cm, (b) +22.59 D, (c) $A_1F = -3.222$ cm, and $A_2F = +3.222$ cm, (d) $A_1H = +1.206$ cm, and $A_2H'' = -1.206$ cm

Solution:

a)

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)t}{nR_1R_2} \right)$$

$$\frac{1}{f} = (1.68 - 1) \left(\frac{1}{5.2} + \frac{1}{-5.2} - \frac{(1.68-1)*3.5}{1.68*-5.2*5.2} \right)$$

$$F = +4.43 \text{ cm}$$

$$\begin{aligned} \text{b) } P &= 1/f(\text{m}) \\ &= +22.57 \text{ D} \end{aligned}$$

c)

$$\begin{aligned} \text{1- first focal point } \quad A_1F_1 &= -f \left(1 - \frac{t}{f_2'} \right) \\ \frac{n'}{f_2'} &= \frac{n' - n}{r_2} \\ \frac{1.68}{f_2'} &= \frac{1.68 - 1}{-5.2} \end{aligned}$$

$$f_2' = -3.22 \text{ cm}$$

$$\text{2- second focal point } \quad A_2F_2 = +f \left(1 - \frac{t}{f_1'} \right)$$

$$\frac{n'}{f_1'} = \frac{n' - n}{r_1}$$

$$\frac{1.68}{f_1'} = \frac{1.68 - 1}{5.2}$$

$$A_2F_2 = +f \left(1 - \frac{t}{f_1'} \right)$$

$$A_2F_2 = +3.222 \text{ cm}$$

d) The principal points.

$$1 - \text{first principle point } A_1H_1 = f \frac{t}{f_2'}$$

$$A_1H = 4.43 \frac{3.5}{12.84}$$

$$A_1H_1 = 1.2068 \text{ cm}$$

$$2 - \text{second principle point } A_2H_2 = -f \frac{t}{f_1'}$$

2-

$$2 - \text{second principle point } A_2H'' = -4.43 \frac{3.5}{12.847}$$

$$A_2H'' = -1.2068 \text{ cm}$$

5.3 A plano-convex lens 2.80 cm thick is made of glass of index 1.530. If the second surface has a radius of 3.50 cm, find (a) the focal length of the lens and (b) the power of the lens. Find the distances from the vertices to (c) the focal points and (d) the principal points.

Solution: another Method

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{(n-1)t}{nR_1R_2} \right)$$

$$\frac{1}{f} = (1.53 - 1) \left(\frac{1}{\infty} + \frac{1}{-3.5} - \frac{(1.53-1)*2.8}{1.53*\infty*-3.5} \right)$$

$$f = -6.605 \text{ cm}$$

$$\begin{aligned} \text{b) } P &= 1/f(\text{m}) \\ &= -15.44 \text{ D} \end{aligned}$$

c) 1 – *first focal point*

$$\begin{aligned} A_1F_1 &= -f \left(1 + \frac{(n-1)t}{nR_2} \right) \\ A_1F_1 &= -6.603 \left(1 + \frac{(1.53-1)*2.8}{1.53*-3.5} \right) \end{aligned}$$

$$= -4.7733 \text{ cm}$$

2 – *the second focal length*

$$\begin{aligned} A_2F_2 \text{ or } A_2F'' &= f \left(1 - \frac{(n-1)t}{nR_1} \right) \\ A_2F_2 &= f \left(1 - \frac{(n-1)t}{nR_1} \right) \end{aligned}$$

$$A_2 F_2 = 6.603 \text{ cm}$$

d) The principal points.

1- The first principle point h_1 or $A_1 H_1$ or

$$A_1 H = -f \frac{(n-1)t}{nR_2}$$

$$A_1 H = -6.603 \frac{(1.53-1) \cdot 2.8}{1.53 \cdot -3.5}$$

$$h_1 \text{ or } A_1 H_1 \text{ or } A_1 H = 1.8298 \\ = 1.83 \text{ cm}$$

2- h_2 The second principle point h_2 or $A_2 H_2$ or

$$A_2 H'' = -f \frac{(n-1)t}{nR_1 \infty}$$

0

$$H_2 \text{ or } A_2 H_2 = 0$$

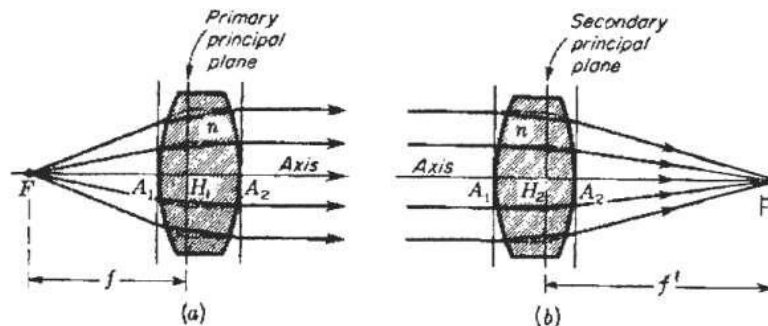


Figure 2. Primary and secondary principal planes of a thick lens. A_1 and A_2 are the vertices.

5.5 A glass lens with radii $r_1 = +2.50 \text{ cm}$ and $r_2 = +4.50 \text{ cm}$ has a thickness of 2.90 cm and an index of 1.630 . Calculate (a) the focal length and (b) the power of the lens.

Find the distances from the vertices to (c) the focal points and (d) the principal points.

Ans. (a) +5.73cm, (b) +17.46D, (e) $A_1F = -7.163$ cm, and $A_2F = +3.162$ cm, (d) $A_1H = -1.433$ cm, and $A_2H'' = -2.568$ cm

5.7 A glass lens with radii $r_1 = +6.50$ cm and $r_2 = +3.20$ cm has a thickness of 2.80 cm and an index of 1.560. Calculate (a) the focal length and (b) the power of this lens in air. Find the distances from the vertices to (c) the focal points and (d) the principal points.

$$a) \frac{1}{f} = (1.560 - 1) \left[\frac{1}{6.5} - \frac{1}{3.2} + \frac{(1.56 - 1) * 2.8}{1.56 * 6.5 * 3.2} \right]$$

$$F = -16.207 \text{ cm}$$

b) the power of this lens in air

$$P = P = 1/f(\text{m}) = -6.223 \text{ Diopter}$$

c) 1 – first focal point

$$A_1F_1 = -f \left(1 + \frac{(n - 1)t}{nR_2} \right)$$

$$A_1F_1 = -(-16.207) \left(1 + \frac{(1.56 - 1) * 2.8}{1.56 * +3.2} \right)$$

$$A_1F_1 = +21.2925 \text{ cm}$$

2 – the second focal length

$$A_2F_2 \text{ or } A_2F'' = f \left(1 - \frac{(n - 1)t}{nR_1} \right)$$

$$A_2F_2 = -16.207 \left(1 - \frac{(1.56 - 1) * 2.8}{1.56 * 6.5} \right)$$

0

$$A_2 F_2 = -13.7008 \text{ cm}$$

d) The principal points.

1- The first principle point h_1 or $A_1 H_1$ or

$$A_1 H = -f \frac{(n-1)t}{nR_2}$$

$$A_1 H = -(-16.603) \frac{(1.53-1)*2.8}{1.53 * -3.5}$$

$$h_1 \text{ or } A_1 H_1 \text{ or } A_1 H = 1.8298$$

$$= 1.83 \text{ cm}$$

2- h_2 The second principle point h_2 or $A_2 H_2$ or

$$A_2 H'' = -f \frac{(n-1)t}{nR_1 \infty}$$

$$H_2 \text{ or } A_2 H_2 = 0$$

0

5.9 A thick lens with radii $r_1 = -4.50 \text{ cm}$ and $r_2 = -3.60 \text{ cm}$ has a thickness of 3.0 cm and an index of 1.560 . Calculate (a) the focal length and (b) the power of the lens.

Find also the distances from the vertices to the corresponding (c) focal points and

(d) principal points.

Ans. (a) + 14.64 cm, (b) +6.83 D, (c) $A_1F = -10.26$ cm, and $A_2P'' = +18.14$ cm,
 (d) $A_1H = +4.38$ cm, and $A_2H'' = +3.502$ cm

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right]$$

where R_1 and R_2 are the radii of the first and the second surfaces.

d : thickness of lens. Fig 2

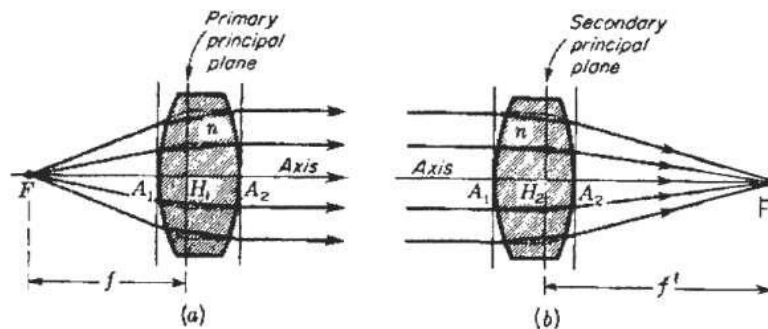


Figure 2. Primary and secondary principal planes of a thick lens. A_1 and A_2 are the vertices.

Step 2: The focal points

The first focal point $A_1F_1 = -f \left[1 + \frac{(n-1)d}{nR_2} \right]$

The second focal point $A_2F_2'' = -f \left[1 - \frac{(n-1)d}{nR_1} \right]$

Step 3: The principal points

The first principal point

$$A_1H_1 = -f \cdot \frac{(n-1)d}{nR_2}$$

PROBLEMS

6.1 A spherical mirror has a radius of - 24.0 cm; An object 3.0 cm high is located in front of the mirror at a distance of (a) 48.0 cm, (b) 36.0 cm, (c) 24.0 cm, (d) 12.0 cm, and (e) 6.0 cm. Find the image distance for each of these object distances.

Ans. (a) + 16.0 cm, (b) + 18.0 cm, (c) + 24.0 cm, (d) + 12.0 cm, (e) - 12.0 cm

Solution

$$\text{a) } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-24}{2}\right) = +12 \text{ cm}$$

$$\frac{1}{48} + \frac{1}{v} = \frac{1}{12}$$
$$v = \frac{u * f}{u - f} = \frac{48 * 12}{48 - 12} = \frac{576}{36} = +16$$

$$\text{b) } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{-24}{2} = -\left(\frac{-24}{2}\right) = +12 \text{ cm}$$

$$\frac{1}{36} + \frac{1}{v} = \frac{1}{12}$$

$$v = \frac{u * f}{u - f} = \frac{36 * 12}{36 - 12} = \frac{432}{24} = +18$$

$$c) \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-24}{2}\right) = +12 \text{ cm}$$

$$\frac{1}{24} + \frac{1}{v} = \frac{1}{12}$$

$$v = \frac{u * f}{u - f} = \frac{24 * 12}{24 - 12} = \frac{288}{12} = +24$$

d) $u=12$; since $f=12$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-24}{2}\right) = +12 \text{ cm}$$

$$\frac{1}{12} + \frac{1}{v} = \frac{1}{12}$$

$$v = \frac{u * f}{u - f} = \frac{12 * 12}{12 - 12} = \frac{144}{0} = \infty$$

$$e) \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-24}{2}\right) = +12 \text{ cm}$$

$$\frac{1}{6} + \frac{1}{v} = \frac{1}{12}$$

$$v = \frac{u * f}{u - f} = \frac{6 * 12}{6 - 12} = \frac{72}{-6} = -12 \text{ cm}$$

6.3 A spherical mirror has a radius of - 15.0 cm. An object 2.50 cm high is located in front of the mirror at a distance of (a) 45.0 cm, (b) 30.0 cm, (c) 15.0 cm, (d) 10.0 cm, and (e) 5.0 cm. Find the image distance for each of these object distances.

Solution

$$a) \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-15}{2}\right) = +7.5 \text{ cm}$$

$$\frac{1}{45} + \frac{1}{v} = \frac{1}{7.5}$$

$$v = \frac{u * f}{u - f} = \frac{7.5 * 45}{45 - 7.5} = \frac{337.5}{37.5} = +9 \text{ cm}$$

$$\text{b) } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-15}{2}\right) = +7.5 \text{ cm}$$

$$\frac{1}{30} + \frac{1}{v} = \frac{1}{7.5}$$

$$v = \frac{u * f}{u - f} = \frac{7.5 * 30}{30 - 7.5} = \frac{225}{22.5} = +10 \text{ cm}$$

$$\text{c) } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-15}{2}\right) = +7.5 \text{ cm}$$

$$\frac{1}{15} + \frac{1}{v} = \frac{1}{7.5}$$

$$v = \frac{u * f}{u - f} = \frac{7.5 * 15}{15 - 7.5} = \frac{112.5}{7.5} = +15 \text{ cm}$$

$$\text{d) } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-15}{2}\right) = +7.5 \text{ cm}$$

$$\frac{1}{10} + \frac{1}{v} = \frac{1}{7.5}$$

$$v = \frac{u * f}{u - f} = \frac{7.5 * 10}{10 - 7.5} = \frac{75}{2.5} = +30 \text{ cm}$$

(e) 5.0 cm

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{-15}{2}\right) = +7.5 \text{ cm}$$

$$\frac{1}{5} + \frac{1}{v} = \frac{1}{7.5}$$

$$v = \frac{u * f}{u - f} = \frac{7.5 * 5}{5 - 7.5} = \frac{37.5}{-2.5} = -15 \text{ cm}$$

6.5 The radius of a spherical mirror is + 18.0 cm. An object 4.0 cm high is located in front of the mirror at a distance of (a) 36.0 cm, (b) 24.0 cm, and (c) 12.0 cm. Find the image distance and image size for each of these object distances.

Ans. (a) -7.20 cm from vertex and +0.80 cm high, *(b)* -6.55 cm from vertex and + 1.092 cm high, *(c)* - 5.40 cm from vertex and + 1.712 cm high.

Solution

a) 36 cm

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{18}{2}\right) = -9 \text{ cm}$$

$$\frac{1}{36} + \frac{1}{v} = \frac{1}{-9}$$

$$v = \frac{u * f}{u - f} = \frac{-9 * 36}{36 + 9} = \frac{-324}{45} = -7.2 \text{ cm}$$

$$m = \frac{-(-7.2)}{36} = +0.2 \leftrightarrow \frac{y'}{y} = \frac{y'}{4}$$

$$y' = 0.8$$

b) u=24 cm

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{18}{2}\right) = -9 \text{ cm}$$

$$\frac{1}{24} + \frac{1}{v} = \frac{1}{-9}$$

$$v = \frac{u * f}{u - f} = \frac{-9 * 24}{24 + 9} = \frac{-216}{33} = -6.545 \text{ cm}$$

$$m = \frac{-(-6.454)}{24} = +0.272 \leftrightarrow \frac{y'}{y} = \frac{y'}{4}$$

$$y' = 1.092$$

c) $u = 12 \text{ cm}$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{18}{2}\right) = -9 \text{ cm}$$

$$\frac{1}{12} + \frac{1}{v} = \frac{1}{-9}$$

$$v = \frac{u * f}{u - f} = \frac{-9 * 12}{12 + 9} = \frac{-108}{21} = -5.1428 \text{ cm}$$

$$m = \frac{-(-5.1428)}{12} = +0.4285 \leftrightarrow \frac{y'}{y} = \frac{y'}{4}$$

$$y' = +1.71428 \text{ cm}$$

6.7 The radius of a spherical mirror is + 8.0 cm. An object 3.50 cm high is located in front of the mirror at a distance of (a) 16.0 cm, (b) 8.0 cm, (c) 4.0 cm, and (d) 2.0 cm.

Find the image distance and image size for each of these object distances.

Solution

a) $u = 16 \text{ cm}$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{8}{2}\right) = -4 \text{ cm}$$

$$\frac{1}{16} + \frac{1}{v} = \frac{1}{-4}$$

$$v = \frac{u * f}{u - f} = \frac{-4 * 16}{16 + 4} = \frac{-64}{20} = -3.2 \text{ cm}$$

$$m = \frac{-(-3.2)}{16} = +0.2 \leftrightarrow = \frac{y'}{y} = \frac{y'}{3.5}$$

$$y' = 0.7 \text{ cm}$$

b) u=8 cm

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{8}{2}\right) = -4 \text{ cm}$$

$$\frac{1}{8} + \frac{1}{v} = \frac{1}{-4}$$

$$v = \frac{u * f}{u - f} = \frac{-4 * 8}{8 + 4} = \frac{-32}{12} = -2.6666 \text{ cm}$$

$$m = \frac{-(-2.666)}{8} = +0.3333 \leftrightarrow = \frac{y'}{y} = \frac{y'}{3.5}$$

$$y' = 1.1666 \text{ cm}$$

c) u= 4 cm

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{8}{2}\right) = -4 \text{ cm}$$

$$\frac{1}{4} + \frac{1}{v} = \frac{1}{-4}$$

$$v = \frac{u * f}{u - f} = \frac{-4 * 4}{4 + 4} = \frac{-16}{8} = -2 \text{ cm}$$

$$m = \frac{-(-2)}{4} = +0.5 \leftrightarrow = \frac{y'}{y} = \frac{y'}{3.5}$$

$$y' = 1.75 \text{ cm}$$

d) $u = 2 \text{ cm}$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \leftrightarrow f = -\frac{r}{2} = -\left(\frac{8}{2}\right) = -4 \text{ cm}$$

$$\frac{1}{2} + \frac{1}{v} = \frac{1}{-4}$$

$$v = \frac{u * f}{u - f} = \frac{-4 * 2}{2 + 4} = \frac{-8}{6} = -1.3333 \text{ cm}$$

$$m = -\frac{v}{u} = \frac{-(-1.3333)}{2} = +0.66666 \leftrightarrow = \frac{y'}{y} = \frac{y'}{3.5}$$

$$y' = 2.333 \text{ cm}$$

6.9 A concave mirror is to be used to focus the image of a tree on a photographic film 8.50 m away from the tree. If a lateral magnification of $-1/20$ is desired, what should be the radius of curvature of the mirror?
 , *Ans.* - 85.2 cm

Solution:

Concave Mirror $f = -r/2$

$u = 8.5 \text{ m}$

$$m = -\frac{v}{u} \leftrightarrow -\frac{1}{20} = -\frac{v}{8.5 \text{ m}}$$

$$v = f = -\frac{r}{2}$$

$$v = f = 42.5 \text{ cm} \leftrightarrow r = -2f = -2 * 42.5 = -85 \text{ cm}$$

6.10 A thin equiconvex lens of index 1.530 and radii of 16.0 cm is silvered on one side.

Find the (a) focal length and (b) power of this system if light enters the unsilvered side.

Solution:

$$\frac{n}{f} = \frac{n' - n}{r_1}$$

$$\frac{1}{f} = \frac{1.53 - 1}{16}$$

$$f = 30.1886 \text{ cm}$$

$$P_1 = \frac{1}{f(m)} = 3.3125 \text{ D}$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 * 1.53 * 100}{-16} = +19.125 \text{ D}$$

$$P = 2P_1 + P_2 \\ = 2 * 3.3125 + 19.125$$

$$P = 25.75 \text{ Diopters}$$

$$P = 1/f(m)$$

$$f = 1/p = 1/25.75 \text{ m} = 3.884 \text{ cm}$$

6.11 A thin lens of index 1.650 has radii' $r_1 = + 5.0 \text{ cm}$ and $r_2 = -15.0 \text{ cm}$.

If the second surface is silvered, what is (a) the focal length and (b) the power of the system?

Solution

$$a) \frac{n}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\frac{n}{f} = (1.65 - 1) \left(\frac{1}{5} + \frac{1}{15} \right)$$
$$f = 5.7 \text{ cm}$$

$$b) P_1 = \frac{n' - n}{r_1} = \frac{1.65 - 1}{0.05} = +13 \text{ D}$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 \cdot 1.65}{-0.15} = +22 \text{ D}$$

$$P = 2P_1 + P_2$$
$$= 2 \cdot 13 + 22$$

$$P = 48 \text{ Diopters}$$

6.12 A thin lens of index 1.720 located in air has radii $r_1 = -6.0$ cm and $r_2 = -12.0$ cm. If the second surface is silvered, what is the power of the system? Use the special-case formulas (6q) and (6r).

$$b) P_1 = \frac{1.72 - 1}{-0.06} = -12 D$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 \cdot 1.72}{-0.12} = +28.66 D$$

$$P = 2P_1 + P_2 \\ = 2 \cdot -12 + 28.66 = -24 + 28.66$$

$$P = 4.66 \text{ Diopters}$$

6.13 A thin lens with a focal length of +10.0 cm is located 2.00 cm in front of a spherical mirror with a radius of -18.0 cm. Find (a) the power, (b) the focal length, (c) the principal point, and (d) the focal point of this thick-mirror optical system.

Ans. (a) +23.11 D, (b) +4.33 cm, (c) $H1H = +2.50$ cm, (d) -1.83 cm

Solution:

$$a) p_1 = 1/f(m) \\ = 1/0.1 = 10 D$$

$$P_2 = \frac{-2n}{r_2} = \frac{-2 \cdot 1}{-0.18} = +11.11 D$$

$$c = \frac{d}{n} = 0.02 m$$

$$a) P = (1 - cP_1)(2P_1 + P_2 - cP_1P_2) \\ = (1 - 0.02 \cdot 10)(2 \cdot 10 + 11.11 - 0.02 \cdot 10 \cdot 11.11) \\ P = 23.11 D$$

b) $f=1/P= +4.33 \text{ cm}$

c) The principal point

$$HH' = \frac{c}{1 - cP_1}$$

$$= \frac{0.02}{1 - 0.02 * 10} = \frac{0.02}{0.8}$$

$HH' = 0.025 = 2.5 \text{ cm}$

d) $AF1 = 2.5 - 4.33$
 $= -1.83$

6.15 A thin lens with a focal length of -12.30 cm is placed 2.50 cm in front of a spherical mirror of radius -9.20 cm. Find the power of

(a) The first lens and (b) the second power.

Calculate (c) the power of the system and (4) its focal length. Locate (e) the principal point and (f) the focal point.

Solution

a) $P_1 = \frac{1}{f(m)} = 1 / -12.3 * 10^{-2}$
 $= -8.13 \text{ D}$

b) $P_2 = \frac{-2n}{r_3} = \frac{-2 * 1}{-9.2 * 10^{-2}} = 21.74 \text{ D}$

$$c = \frac{d}{n} = \frac{0.025}{1} = 0.025 \text{ m}$$

c) $P = (1 - c P_1)(2P_1 + P_2 - c P_1 P_2)$

$$P = (1 - 0.025 * -8.13) (2 * -8.13 + 21.74 - 0.025 * -8.13 * 21.74)$$

$P = 11.9 \text{ D}$

d) $f = \frac{1}{P} = 8.403 \text{ cm}$

$$e) HH' = \frac{c}{1 - c P_1} = \frac{0.025}{1 - 0.025 * -8.13}$$

$$HH' = 2.07 \text{ cm}$$

$$F.F.l = f - HH' = 8.43 - 2.07 = 6.35 \text{ cm}$$

6.17 A thick lens of index 1.560 has radii $r_1 = +15.0 \text{ cm}$ and $r_2 = -30.0 \text{ cm}$. If the second surface is silvered and the lens is 5.0 cm thick, find (a) the power, (b) the focal length, (c) the principal point, and (d) the focal point.

Ans. (a) +14.67 D, (b) +6.82 cm, (c) $H_1H = +3.640 \text{ cm}$, (d) $H_1F = +3.180 \text{ cm}$

Solution

$$P_1 = \frac{n' - n}{r_1} = \frac{1.56 - 1}{0.15} = +3.7 \text{ D}$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 * 1.56}{-0.3} = +10.4 \text{ D}$$

$$c = \frac{d}{n'} = \frac{.05}{1.56} = 0.032 \text{ m}$$

$$\begin{aligned} a) P &= (1 - c P_1)(2P_1 + P_2 - c P_1 P_2) \\ &= (1 - 0.032 * 3.7)(2 * 3.7 + 10.4 - 0.032 * 3.7 * 10.4) \\ P &= 14.67 \text{ D} \end{aligned}$$

b) the focal length

$$f = \frac{1}{P} = \frac{1}{14.67} = 6.8166 \text{ cm}$$

c) the principle point

$$HH' = \frac{c}{1 - c P_1} = \frac{0.032}{1 - 0.032 * -3.7} = 3.6297 \text{ cm}$$

$$H_1F = f - HH' = 6.849 - 3.6297 = +3.18 \text{ cm}$$

6.19 A lens 4.50 cm thick has an index of 1.720 and radii

$r_1 = -6.0$ cm and, $r_2 = -12.0$ cm. If the second surface is silvered, find (a) the power, (b) the focal length, (c) the position of the principal point, and (d) the position of the focal point.

Solution

$$P_1 = \frac{n' - n}{r_1} = \frac{1.72 - 1}{-0.06} = -12 D$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 \cdot 1.72}{-0.12} = +28.666 D$$

$$c = \frac{d}{n'} = \frac{0.045}{1.72} = 0.02616 m$$

$$\begin{aligned} \text{a) } P &= (1 - c P_1)(2P_1 + P_2 - c P_1 P_2) \\ &= (1 - 0.02616 * -12)(2 * -12 + 28.666 - .02616 * -12 \\ &\quad * 28.666) \\ P &= 9.375145 D \end{aligned}$$

b) the focal length

$$f = \frac{1}{P} = \frac{1}{9.375145} = 10.666 \text{ cm}$$

c) the principle point

$$\begin{aligned} \text{HH}' &= \frac{c}{1 - c P_1} = \frac{0.02616}{1 - 0.02616 * -12} = 3.812 \text{ cm} \\ \text{H}_1\text{F} &= f - \text{HH}' = 10.666 - 3.812 = +6.854 \text{ cm} \end{aligned}$$

6.21 The curved surface of a plano-convex lens has a radius of 20.0 cm. The refractive index of the glass is 1.650, and the thickness is 2.750 cm. If the curved surface is silvered, find (a) the power, (b) the focal length, (c) the principal point, and (d) the focal point.

Ans. (a) + 16.50 D, (b) + 6.06 cm, (c) + 1.667 cm, (d) +4.394 cm

Solution

Solution

$$P_1 = \frac{n' - n}{r_1} = 0$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 \cdot 1.65}{-0.2} = +16.5 D$$

$$c = \frac{d}{n'} = \frac{0.0275}{1.65} = 0.01667 m$$

$$a) P = (1 - c P_1)(2P_1 + P_2 - c P_1 P_2)$$

$$P = 16.5 D$$

b) the focal length

$$f = \frac{1}{P} = \frac{1}{16.5} = 6.06 cm$$

c) the principle point

$$HH' = \frac{c}{1 - c P_1} = \frac{0.01667}{1 - 0} = 1.667 cm$$

$$H_1F = f - HH' = 6.06 - 1.667 = +4.394 cm$$

$$P_1 = \frac{n' - n}{r_1} = 0$$

$$P_2 = \frac{-2n'}{r_2} = \frac{-2 \cdot 1.65}{-0.2} = +16.5 D$$

$$c = \frac{d}{n'} = \frac{0.0275}{1.65} = 0.01667 m$$

$$d) P = (1 - c P_1)(2P_1 + P_2 - c P_1 P_2)$$

$$P = 16.5 D$$

e) the focal length

$$f = \frac{1}{P} = \frac{1}{16.5} = 6.06 \text{ cm}$$

f) the principle point

$$HH' = \frac{c}{1-c P_1} = \frac{0.01667}{1-0} = 1.667 \text{ cm}$$

$$H_1F = f - HH' = 6.06 - 1.667 = +4.394 \text{ cm}$$

6.23 If the plane surface of the lens given in Prob. 6.21 is silvered in plac