∴ chapter one ∴

∴ Distribution function of Random variable ∴

1_ The Moment Generating function method.

Def:-
let X have the pdf $f(x)$ and the Mgf $M_x(t)$
Then $y = g(x)$ has density fun. Given as:-

$$M_y(t) = E(e^{yt})$$

Ex) let $X_i \sim \chi^2(1)$, $i = 1, 2, 3$. are independent r.v.s

let $y = X_1 + X_2 + X_3$, Find dist. of Y by Mgf method.

Sol) $y = X_1 + X_2 + X_3$, $X_i \sim \chi^2(1)$

$$M_x(t) = (1-2t)^{-\frac{r}{2}}$$

$$\because r = 1 \Rightarrow M_x(t) = (1-2t)^{-\frac{1}{2}}$$

القانون $\rightarrow$ $M_y(t) = E(e^{yt}) = E(e^{(X_1+X_2+X_3)t})$

$$= E(e^{X_1 t + X_2 t + X_3 t})$$

$$= E(e^{X_1 t}) . E(e^{X_2 t}) . E(e^{X_3 t})$$

$$= (1-2t)^{-\frac{1}{2}} . (1-2t)^{-\frac{1}{2}} . (1-2t)^{-\frac{1}{2}}$$

$$= (1-2t)^{-\frac{3}{2}}$$

$\therefore Y \sim \chi^2(3)$

Ex2) let $X_i \sim Ber(\theta)$, $i=1,2,3,4$. are independe
r.v.s, let $Y = X_1 + X_2 + X_3 + X_4$

Find dist of y by using Mgf method.

sol)

$$M_y(t) = E(e^{yt}) = E(e^{(x_1+x_2+x_3+x_4)t})$$

$$= E(e^{x_1t + x_2t + x_3t + x_4t})$$

$$= E(e^{x_1t}) . E(e^{x_2t}) . E(e^{x_3t}) . E(e^{x_4t})$$

$$= M_{x_1}(t) . M_{x_2}(t) . M_{x_3}(t) . M_{x_4}(t).$$

$\because X_i \sim Ber(\theta)$

$$M_x(t) = ((1-\theta) + \theta e^t)$$

$$\therefore M_y(t) = ((1-\theta) + \theta e^t)^4$$

$\therefore Y \sim Bin(4, \theta)$

$X \sim Bin(n, \theta) \rightarrow M_x(t) = ((1-\theta) + \theta e^t)^n$

Ex3) let $X \sim b(n,p)$ is independent r.v and let $Y = n - X$. Find dist of y by using Mgf method.

Sol) $\because X \sim b(n,p)$

From Table: $X \sim Bin(n,\theta) \to M_X(t) = ((1-\theta)+\theta e^t)^n$

$\Rightarrow X \sim b(n,p) \to M_X(t) = ((1-p)+pe^t)^n$

تطبيق لقانون $\to M_Y(t) = E(e^{Yt}) = E(e^{(n-X)t}) = E(e^{nt-Xt})$

$= e^{nt} \cdot E(e^{-Xt})$

constant

$= e^{nt} \cdot M_X(-t)$

$= e^{nt} \cdot ((1-p)+pe^{-t})^n$

$= (e^t \cdot ((1-p)+pe^{-t}))^n$

$= ((1-p)e^t + p)^n$

$M_Y(t) = (p + (1-p)e^t)^n$

$\therefore Y \sim b(n, 1-p)$

H.W) 1- let $X \sim beta(2,3)$, if $Y = Ln(x)$, Find $M_Y(t)$

2- let $X_i \sim Pos(\lambda_i)$, $i=1,2$. are independent r.v.s

Find the dist. of y if $y = x_1 - x_2$ and $z = x_1 + x_2$