Transformation

∴ Transformation for discrete case ∴

Def)

let X have the p.d.f. $f(x)$, Then $y=h(x)$ has density fun. Given as;-

$$\begin{aligned} g(y) &= P(Y=y) \\ &= f(h(x)=y) \\ &= f(x=w(y)) \end{aligned}$$

Ex1) let X be a r.v having with

$$f(x)=\begin{cases} \frac{e^{-\lambda}\cdot\lambda^{x}}{x!} & , x=0,1,\dots \\ 0 & , \text{o.w.} \end{cases}$$

Determine the P.d.f of $y=4x$.

Sol) $y=4x$

$$\rightarrow g(y)=P(Y=y)=P(4x=y)=P\left(x=\tfrac{1}{4}y\right)$$

$$g(y)=P\left(x=\tfrac{1}{4}y\right)=\begin{cases} \frac{e^{-\lambda}\cdot\lambda^{\frac{1}{4}y}}{(\frac{1}{4}y)!} & , y=0,4,8,\dots \\ 0 & , \text{o.w} \end{cases}$$

$A=\{0,1,2,\dots\}$, $y=4x \rightarrow \begin{matrix} x=0\rightarrow y=0 \\ x=1\rightarrow y=4 \end{matrix}$

$B=\{0,4,8,\dots\}$

Ex2) let X have a Binomal dist. with $n=6$, $P=\frac{1}{3}$. Determine the P.d.f of $y=x^2$ by Transformation method.

Sol) $\because X \sim Bin(n,\theta)$, $\theta = P$

$$f(x)=\begin{cases} C_x^n \theta^x (1-\theta)^{n-x} & , x=0,1,\dots,n \\ 0 & , o.w \end{cases}$$

$y=x^2 \rightarrow x=\pm\sqrt{y}$

$g(y)=P(Y=y)=P(x=\pm\sqrt{y})$

$\because n=0,1,\dots,6$, $P=\frac{1}{3}$

$$f(x)=\begin{cases} C_x^6 P^x (1-P)^{6-x} & , x=0,1,\dots,6 \\ 0 & , o.w \end{cases}$$

$$\therefore g(y)=P(x=\sqrt{y})=\begin{cases} C_{\sqrt{y}}^6 \left(\frac{1}{3}\right)^{\sqrt{y}} \left(1-\frac{1}{3}\right)^{6-\sqrt{y}} & , y=0,\dots,36 \\ 0 & , o.w \end{cases}$$

H.W

1- if $X \sim P(\theta^2)$ Find $g(y)$, if $y=(e^x-4)$ by Transformation method.

2- let X have a binomial dist. with $n=6$, $P=\frac{1}{3}$. Find P.d.f of y by Using Transformation if $y=x+1$.