"Transformation"

∴ Transformation For Continuous Case ∴

Def) let X be continuous r.v. with P.d.f. $f(x)$

Then $Y = h(x)$ has density fun. Given as:-

احفظ
$$\boxed{g(y) = f(x = w(y)) \cdot |J|}$$

where J is from Jacobin and absolute value

of $J = |J| = \left|\frac{dx}{dy}\right|$

Ex) if $f(x) = \begin{cases} 2x &, 0<x<1 \\ 0 &, o.w \end{cases}$

Find P.d.f of Y when 1) $Y = -4X+3$

2) $Y = 8X^3$

Sol)

$$g(y) = f(x = \underbrace{w(y)}_{?}) \cdot \underbrace{|J|}_{?}$$

1)

$Y = -4X + 3 \rightarrow [+4X = 3 - Y] \div 4$

$\rightarrow X = \dfrac{3-Y}{4}$, $f(x) = \begin{cases} 2x &, 0<x<1 \\ 0 &, o.w \end{cases}$

$0<x<1 \longrightarrow Y = -4x+3$

اخوطها $\rightarrow x = 0 \rightarrow Y = -4 \times 0 + 3$

$Y = 3$

$x = 1 \rightarrow \quad y = -4 \times 1 + 3 \Rightarrow y = -1$

$\therefore \quad -1 < y < 3$

$\therefore \quad x = \frac{3}{4} - \frac{y}{4} \quad , \quad |J| = \left|\frac{dx}{dy}\right|$

$|J| = \left|\frac{-1}{4}\right| = \frac{1}{4}$

$\therefore g(y) = f(x = w(y)) \cdot |J|$

$= 2((3-y)/4) \cdot (\frac{1}{4})$

from $\rightarrow f(x) = \begin{cases} 2x & , 0 < x < 1 \\ 0 & , o.w \end{cases}$

$\therefore g(y) = \begin{cases} (3-y)/8 & , -1 < y < 3 \\ 0 & , o.w \end{cases}$

2) since $y = 8x^3$, $f(x) = \begin{cases} 2x & , 0 < x < 1 \\ 0 & , o.w \end{cases}$

$x^3 = \frac{y}{8} \Rightarrow x = \sqrt[3]{\frac{y}{8}} = \frac{y^{1/3}}{2}$

$|J| = \frac{1}{6} y^{-2/3}$, $x = 0 \rightarrow y = 8(0)^3 = 0$

$x = 1 \rightarrow y = 8(1)^3 = 8$

$\therefore 0 < y < 8$

$\therefore g(y) = f(x = w(y)) \cdot |J|$

$\rightarrow g(y) = \{ \frac{1}{6} y^{-\frac{1}{3}} , \quad 0 < y < 8 \}$

∴ Transformation ∴

Ex2) if $X \sim N(0,1)$. Find $g(y)$ by Transformation method if $Y = |X|$.

Sol)

$$X \sim N(0,1) \Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, & -\infty < x < \infty \\ 0, & o.w \end{cases}$$

$$g(y) = f(x = w(y)) \cdot |J|$$

$\rightarrow Y = |X| \Rightarrow$ by Transformation $x = \pm y$

$$\Rightarrow |J| = \left|\frac{dx}{dy}\right| = |\pm 1| = 1$$

$$\rightarrow g(y) = f(x = +y) \cdot |+1| + f(x = -y) \cdot |-1|$$

$$\Rightarrow f(x = +y) = \left\{ \begin{matrix} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, & 0 < y < \infty \\ 0, & o.w \end{matrix} \right\}$$

$$f(x = -y) = \left\{ \begin{matrix} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, & 0 < y < \infty \\ 0, & o.w \end{matrix} \right\}$$

$\because -\infty < x < \infty \rightarrow y = |x|$ $\rightarrow D_f = \mathbb{R}$, $\rightarrow R_f = \{y : 0 \leq y < \infty$ $[0, +\infty)$

∴ Transformation ∴

$$g(y) = \begin{cases} \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} , & 0 < y < \infty \\ 0 . & o.w \end{cases}$$

Ex3) if $X \sim Beta(a,b)$, Find $g(y)$ by using Transformation method if $y = 1 + x$.

Sol) $y = 1 + x \rightarrow x = y - 1$

$$|J| = \left|\frac{dx}{dy}\right| = |1| = 1$$

$\because X \sim Beta(a,b) \Rightarrow f(x) = \begin{cases} \frac{1}{\beta(a,b)} x^{a-1} (1-x)^{b-1}, & 0 \leq x \leq 1 \\ 0, & o.w \end{cases}$

$$\Rightarrow g(y) = f(x = w(y)) \, |J|.$$

$$= \frac{1}{\beta(a-b)} \cdot (y-1)^{a-1} \cdot (1-(y-1))^{b-1} \cdot |1|$$

$$\Rightarrow g(y) = \begin{cases} \frac{1}{\beta(a-b)} \cdot (y-1)^{a-1} \cdot (2-y)^{b-1} , & 1 < y < 2 \\ 0, & o.w \end{cases}$$

H.w if $f(x) = \begin{cases} 2x e^{-x^2} , & x > 0 \\ 0 , & o.w \end{cases}$.

Find p.d.f of y if $y = x^2$.