* بعض العلاقات لتوزيعات دوال المتغيرات العشوائية *

1- if $Z \sim N(0,1)$ Then $Y = Z^2 \sim \chi^2(1)$

2- if $U_i \sim \chi^2(r_i)$, $i = 1, 2, \dots, K$. are indep. r.v.s

Then $y = \sum_{i=1}^{K} U_i \sim \chi^2\left(\sum_{i=1}^{K} r_i\right)$

3- if $Z \sim N(0,1)$ and $U \sim \chi^2(r)$ where Z and U are indep. Then

$$T = \frac{Z}{\sqrt{\frac{U}{r}}} \sim t(r)$$

4- if $U \sim \chi^2(r_1)$, and $v \sim \chi^2(r_2)$ are indep. r.v.s

Then $F = \frac{U/r_1}{V/r_2} \sim F(r_1, r_2)$

5- if $X \sim Ber(\theta)$ Then $Y = \sum_{i=1}^{n} x_i \sim Bin(n, \theta)$

6- if $X \sim Bin(n, \theta)$ Then $y = \sum_{i=1}^{m} x_i \sim Bin(nm, \theta$

7- if $x \sim P(\theta)$ Then $Y = \sum_{i=1}^{n} x_i \sim P(n\theta)$

8- if $X \sim N(\mu, \sigma^2)$ Then $y = \sum_{i=1}^{n} x_i \sim N(n\mu, n\sigma^2)$

9- if $X \sim N(\mu, \sigma^2)$ then

$$y = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

10- if $X \sim Exp(\theta)$ Then $Y = \sum_{i=1}^{n} X_i \sim Gam(n, \theta)$

11- if $X \sim Geo(\theta)$ Then $Y = \sum_{i=1}^{n} X_i \sim NB(n, \theta)$

12- if $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1, 2, \dots, n$. are indep. r.v.s
Then $Y = \sum_{i=1}^{n} a_i X_i \sim N\left(\sum_{i=1}^{n} a_i \mu_i, \sum_{i=1}^{n} a_i^2 \sigma_i^2\right)$

13- if $X \sim N(\mu, \sigma^2)$ then $Z = \frac{X - \mu}{\sqrt{\sigma^2}} \sim N(0,1)$.

Ex1) if $Z \sim N(0,1)$ and $Y = Z^2$, find dist. of Y?

Sol) since $Z \sim N(0,1)$

Then $Y = Z^2 \sim \chi^2(1)$

Ex2) if $U \sim \chi^2(5)$ and $V \sim \chi^2(8)$ are indep. r.v.s, find distribution of $W = \frac{U/5}{V/8}$

Sol) From Point (4) $U \sim \chi^2(r_1)$, $V \sim \chi^2(r_2)$
$r_1 = 5$, $r_2 = 8$

$$W = \frac{U/r_1}{V/r_2} \sim F(r_1, r_2) = \frac{U/5}{V/8} \sim F(5,8)$$

Ex3) if $X_1 \sim \chi^2(4)$, $X_2 \sim \chi^2(3)$ and $X_3 \sim \chi^2(1)$ are independent r.v. Find

1) dist. of $y = \sum_{i=1}^{3} X_i$

2) $E(6y-1)$ 3) $V(2y-3)$

Sol)

1) dist. of $y = \sum_{i=1}^{3} X_i$

From point (2) $y = \sum_{i=1}^{k} U_i \sim \chi^2(\sum_{i=1}^{k} r_i)$

$r_1 = 4$, $r_2 = 3$, $r_3 = 1$

$\sum_{i=1}^{3} r_i = 4+3+1 = 8 \quad \therefore Y \sim \chi^2(8)$

2) $E(6y-1) \rightarrow E(Y) = 8$, $V(Y) = 2(8) = 16$

$E(6y-1) = 6E(y) - 1 = 6(8) - 1 = 47$

3) $V(2y-3) = 2^2 V(Y) = 4(16) = 64$

(H.W) 1- if $X_1 \sim N(1,2)$, $X_2 \sim N(1.5, 47/4)$ $X_3 \sim N(0,3)$ are indep. r.v.s and let $A = 3X_1 + 2X_2 + 5X_3$, find dist for A.

2- if $Z_i \sim N(0,1)$, $i = 1, \dots, 5$, are indep. r.v.s find dist of $y = \dfrac{Z_1}{\sqrt{(Z_2^2 + Z_3^2 + Z_4^2)/3}}$