Chapter one

Functions الدوال

Definition:

Let A and B be non-empty sets. The relation f from A into B is called function, if each element of A is f- related to one and only one element of B.

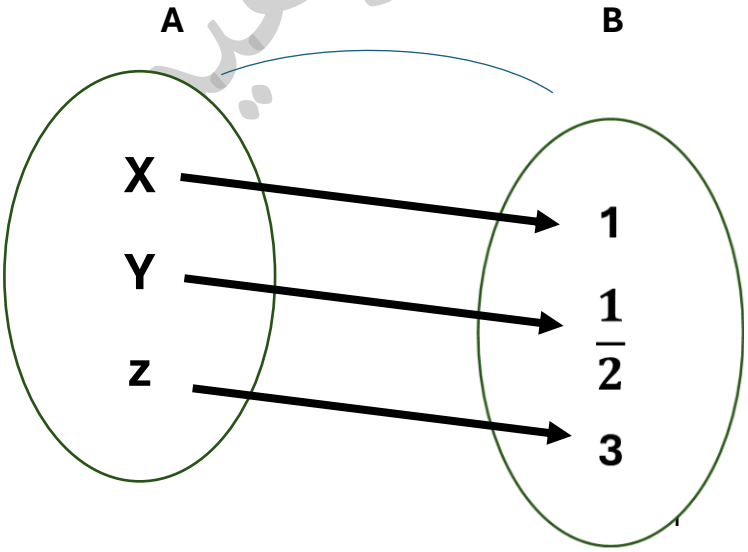
i.e

$$\forall\, x \in A\ \exists!\ y \in B \qquad s.t$$
$$f(x) = y$$

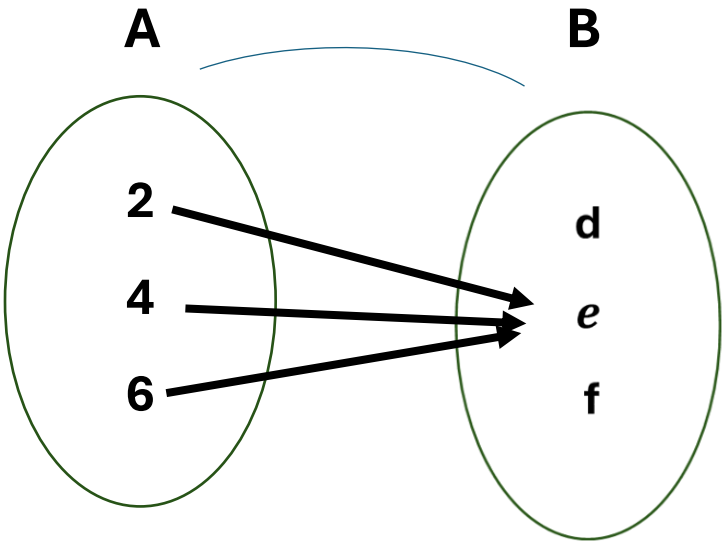
هي علاقة تقرن كل عنصر ينتمي الى مجالها بعنصر واحد وواحد فقط في مجالها المقابل وتكتب الدالة f تكون

$$f(x) = y$$

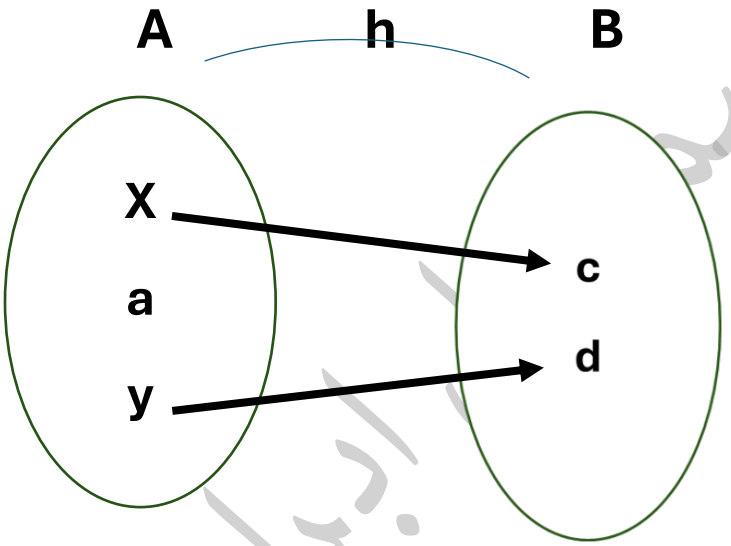
For Examples



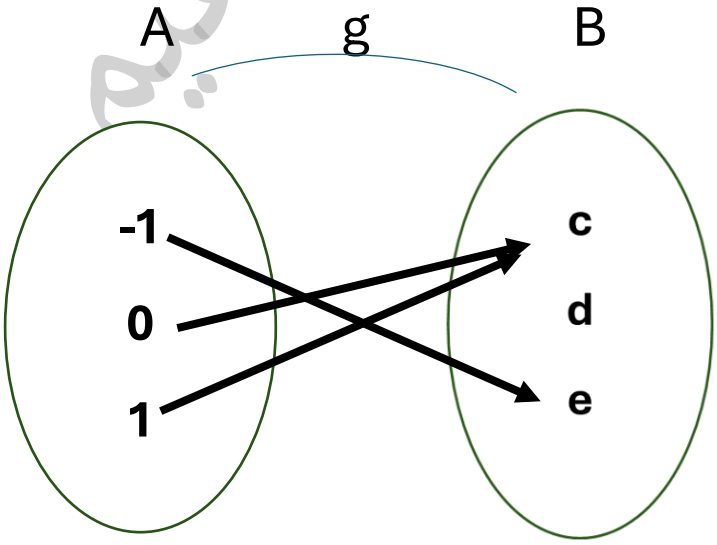
Is a function.



Is a function.



Is not a function.



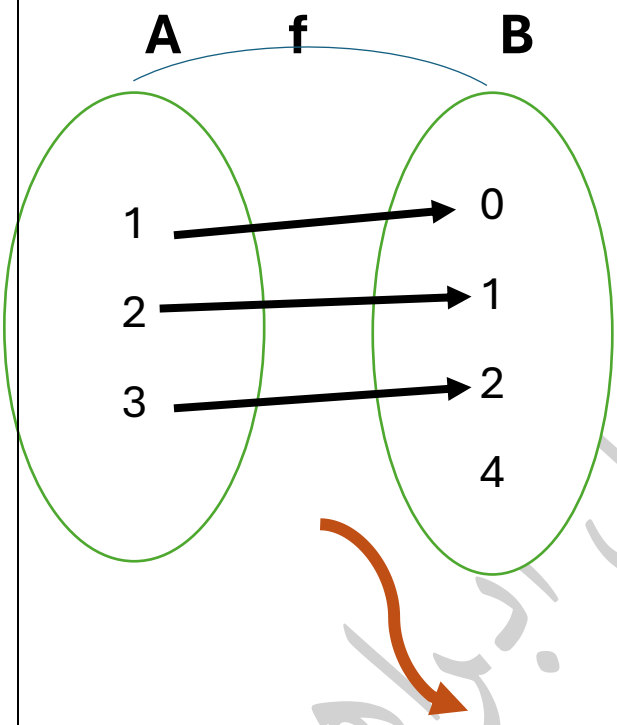
Is a function.

Definition:

A function $f: A \to B$ is called onto (surjective) if and only if

$$\forall\, b \in B \quad \exists\, a \in A \quad s.t$$

$$f(a) = b$$



الدالة $f: A \to B$ تسمى دالة متعددة اذا كان كل عنصر في B هو صورة لأحد عناصر A

الدالة f ليست onto لان 4 ليس لها صورة

Example:

$$f(x) = 2x + 1 \qquad \ldots(1)$$

Let $y \in f$

❖ $y = 2x + 1$

$y - 1 = 2x$

→ $x = \frac{y-1}{2} \qquad \ldots(2)$

Sub in eq (1)

$$f(x) = 2\left(\frac{y-1}{2}\right) + 1$$

$$f(x) = y$$

الدالة متعددة

Definition:

$f: A \to B$ is called one to one(injective) iff

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$$\forall x_1, x_2 \in A$$

Example:

$$f(x) = \frac{x-2}{x+1}$$

Let $f(x_1) = \frac{x_1-2}{x_1+1}$, $f(x_2) = \frac{x_2-2}{x_2+1}$

Now to prove f is one to one

$$\text{Let } f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1-2}{x_1+1} = \frac{x_2-2}{x_2+1}$$

$$(x_1+1)(x_2-2) = (x_1-2)(x_2+1)$$

$$x_1x_2 - 2 - 2x_1 + x_2 = x_1x_2 + x_1 - 2x_2 - 2$$

$$-x_1 - 2x_1 = -x_2 - 2x_2$$

$$-3x_1 = -3x_2$$

$$\Rightarrow \quad x_1 = x_2 \quad , \text{ thus } f \text{ 1-1}$$

Example:

$$f(x) = 2x^3 - 7$$

Let $f(x_1) = 2x_1^3 - 7$, $f(x_2) = 2x_2^3 - 7$

Now to prove f is one to one

$$\text{Let } f(x_1) = f(x_2)$$

$$\Rightarrow 2x_1^3 - 7 = 2x_2^3 - 7$$

$$\Rightarrow 2x_1^3 = 2x_2^3$$

$$\Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow \quad x_1 = x_2 \quad , \text{ thus } f \text{ 1-1}$$

Definition:

A function $f: A \to B$ is called bijective iff f is both one to one (1-1) and onto.

Example:

$$f: A \to B \qquad \text{s.t} \qquad f(x) = x$$

To prove f is bijective we must prove f is one to one and onto

1- $\forall x \in B \quad \exists x \in A \quad$ s.t

$f(x) = x$

f is onto.

2- If $x_1 = x_2$

$\therefore f(x_1) = f(x_2)$

Thus, f is one to one

Therefore, by (1) and (2) we get f is bijective.

H.W

Is f is one to one or onto or bijective in the following

1- $f(x) = x - 5$

2- $f(n) = 2n$

3- $f(x) = 5x - 2$

4- $f(x) = \frac{x-2}{x+1}$