Gauss_Seidel Method

1- $\max \sum_{j=1}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1$ ← شرط التقارب

شرط التوقف

2- $| x_i^{(k+1)} - x_i^{(k)} | \leq \epsilon$

3- $x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$

$i = 1, 2, ---, n$ ← الصيغة العامة

Ex:- By using the Gauss_Seidel Method Find the solution of the following systems?

$4x_1 + x_2 - 2x_3 = 1$
$x_1 + 3x_2 - x_3 = 8$
$x_1 - 7x_2 + 10x_3 = 2$

"use Two loops" where $(x_1^{(0)} = 1,\ x_2^{(0)} = 3,\ x_3^{(0)} = 2)$

Solution :-

استخدام شرط التقارب

$\max \sum_{j=1}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1$

$4x_1 + x_2 - 2x_3 = 1 \rightarrow a_{11} = 4 \rightarrow \left|\frac{1}{4}\right| + \left|\frac{-2}{4}\right| = 0.75$

$x_1 + 3x_2 - x_3 = 8 \rightarrow a_{22} = 3 \rightarrow \left|\frac{1}{3}\right| + \left|\frac{-1}{3}\right| = 0.66$

$x_1 - 7x_2 + 10x_3 = 2 \rightarrow a_{33} = 10 \rightarrow \left|\frac{1}{10}\right| + \left|\frac{-7}{10}\right| = 0.8$

$\max \{0.75, 0.66, 0.8\} = 0.8 < 1$

∴ الترتيب صحيح

∴ Gauss-Seidel Method ∴

$4x_1 + x_2 - 2x_3 = 1 \rightarrow x_1 = \frac{1}{4}(1 - x_2 + 2x_3)$

$x_1 + 3x_2 - x_3 = 8 \rightarrow x_2 = \frac{1}{3}(8 - x_1 + x_3)$

$x_1 - 7x + 10x_3 = 2 \rightarrow x_3 = \frac{1}{10}(2 - x_1 + 7x_2)$

$x_1^{(0)} = 1 \ , \ x_2^{(0)} = 3 \ , \ x_3^{(0)} = 2$

$k=0 \rightarrow \quad x_1^{(k+1)} = \frac{1}{4}(1 - x_2^{(k)} + 2x_3^{(k)})$

$x_1^{(1)} = \frac{1}{4}(1 - 3 + 4) = 0.5$

$x_2^{(k+1)} = \frac{1}{3}(8 - x_1^{(1)} + x_3^{(0)})$

$= \frac{1}{3}(8 - (0.5) + 2) = 3.166$

$x_3^{(k+1)} = \frac{1}{10}(2 - x_1^{(1)} + 7x_2^{(1)})$

$= \frac{1}{10}(2 - 0.5 + 7(3.166))$

$= 2.367$

$x_1^{(1)} = 0.5 \ , \ x_2^{(1)} = 3.166 \ , \ x_3^{(1)} = 2.367$

→ Two loops $\quad k=1$

$x_1^{(2)} = \frac{1}{4}(1 - x_2^{(1)} + 2x_3^{(1)}) = \frac{1}{4}(1 - 3.166 + 2(2.367))$

$= \boxed{0.642}$

$x_2^{(2)} = \frac{1}{3}(8 - x_1^{(2)} + x_3^{(1)}) = \frac{1}{3}(8 - 0.642 + 2.367) = \boxed{3.242}$

$x_3^{(2)} = \frac{1}{10}(2 - x_1^{(2)} + 7x_2^{(2)}) = \frac{1}{10}(2 - (0.642) + 7(3.242))$

$\boxed{x_3 = 2.405}$