∴ Interpolation theory ∴

- 1- Lagrange Interpolation Polynomials
- 2- Finite Difference
- 3- Divided finite difference

1- Lagrange Interpolation Polynomials;-

x	x_0	x_1	x_2
$f(x)$	$f(x_0)$	$f(x_1)$	$f(x_2)$

$$f(x^*) = \sum_{j=0}^{n} f(x_j)\, L_i(x)$$

$$L_i(x) = \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{(x^* - x_i)}{(x_j - x_i)} \quad , i = 0, 1, 2, \ldots$$

For Example :- $n = 2$, x_0, x_1, x_2

$$= f(x_0) L_0(x^*) + f(x_1) L_1(x^*) + f(x_2) L_2(x^*$$

$j = 0$, $i = 1, 2$

$$f(x^*) = \frac{(x^* - x_1) \cdot (x^* - x_2)}{(x_0 - x_1) \cdot (x_0 - x_2)} \cdot f(x_0)$$

$j = 1$, $i = 0, 2$ $$+ \frac{(x^* - x_0) \cdot (x^* - x_2)}{(x_1 - x_0) \cdot (x_1 - x_2)} \cdot f(x_1)$$

$j = 2$, $i = 0, 1$ $$+ \frac{(x^* - x_0) \cdot (x^* - x_1)}{(x_2 - x_0) \cdot (x_2 - x_1)} \cdot f(x_2)$$

∴ Lagrange Interpolation ∴
∴ Polynomials ∴

Ex:- Find F(2), From the following data base by using lagrange algorithm.

x	1.1	1.7	3
F(x)	10.6	13.2	20.3

$$F(x^*) = \sum_{j=0}^{n=2} F(x_j) \cdot L_i(x) = F(x_0) \cdot L_0(x^*) + F(x_1) \cdot L_1(x^*) + F(x_2) \cdot L_2(x^*)$$

$j = 0, \; i = 1, 2$

$$F(2) = \frac{(x^* - x_1)\cdot(x^* - x_2)}{(x_0 - x_1)\cdot(x_0 - x_2)} \cdot F(x_0)$$

$j = 1$

$$+ \frac{(x^* - x_0)\cdot(x^* - x_2)}{(x_1 - x_0)\cdot(x_1 - x_2)} F(x_1) + \frac{(x^* - x_0)\cdot(x^* - x_1)}{(x_2 - x_0)\cdot(x_2 - x_1)} F(x_2)$$

$$\Rightarrow F(2) = \frac{(2 - 1.7)\cdot(2-3)}{(1.1 - 1.7)\cdot(1.1-3)} \cdot (10.6) \quad , j = 0, \; i = 1, 2$$

$$+ \frac{(2-1.1)\cdot(2-3)}{(1.7 - 1.1)\cdot(1.7-3)} \cdot (13.2) + \frac{(2-1.1)\cdot(2-1.7)}{(3-1.1)\cdot(3-1.7)} \cdot (20.3$$

$$\Rightarrow F(2) = 14.644$$

Ex:- By Lagrange Formula Find the value of the Function $f(3)$.

	0	1	2	4
)	1	1	2	5

olution:-

$$P(x^*) = \frac{(x^*-x_1)(x^*-x_2)(x^*-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot P(x_0) + \dots$$

$$P(3) = \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)} \cdot (1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)} (1)$$

$$+ \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)} (2) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)} (5)$$

$$= 0.25 - 1 + 3 + 1.25$$

$$= 3.5$$

2-

x	1	2	4	5
f(x)	16	96	88	0

Find $f(3)$

$$f(x^*) = \frac{(x^*-x_1)(x^*-x_2)(x^*-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot f(x_0) + \dots$$

$$f(3) = \frac{(3-2)(3-4)(3-5)}{(1-2)(1-4)(1-5)} \cdot (16)$$

$$+ \frac{(3-1)(3-4)(3-5)}{(2-1)(2-4)(2-5)} \cdot (96) + \frac{(3-1)(3-2)(3-5)}{(4-1)(4-2)(4-5)} (88)$$

$$+ \frac{(3-1)(3-2)(3-4)}{(5-1)(5-2)(5-4)} \cdot (0)$$

$$= -2.666 + 64 + 58.666$$

$$= 120$$