∴ Order Statistics ∴

Distribution of the k-th order statistic

$$Y_{(k)} = X_{(k)}$$

Suppose $X_1, X_2, \dots, X_n$ are i.i.d random variables with common distribution function $F_x(x)$ and common densit[y] function $f_x(x)$.

- The density function of $X_{(k)}$ or $Y_{(k)}$ is:-

$$g(y_k) = \frac{n!}{(k-1)!\,(n-k)!} \cdot \left(F_x(y_k)\right)^{k-1} \cdot \left(1 - F_x(y_k)\right)^{n-k} \cdot f_x(y_k).$$

P.d.f of the smallest order statistics

- if $X_1, X_2, \dots, X_n$ be a r.s of size n from a populati[on] with continuous p.d.f $f(x)$, then the p.d.f of the smallest order statistics $X_{(1)}$ is given as:-

$$g(y_1) = n\left(1 - F_x(y_1)\right)^{n-1} \cdot f_x(y_1)$$

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Example:-

let $X_i \sim Exp(1);\ i=1,\dots,5$ and $y_i;\ i=1,\dots,5$ be OS,

find $g(y)$ where $y=\min(x_i)$

Sol

$\because$ Find $g(y) \longrightarrow y=\min(x_i)$

$\therefore$ smallest order statistic

$\boxed{n=5}$

$$g(y_1)=n\,(1-F_x(y_1))^{n-1}\cdot f_x(y_1)$$

بالتعويض

$$g(y_1)=5\,(1-F_x(y_1))^{4}\cdot f_x(y_1)$$

$$\because X_i \sim Exp(1) \longrightarrow f(x)=\begin{cases} e^{-x} & , x>0 \\ 0 & , o.w. \end{cases}$$

$$\Rightarrow f_x(y_1)=e^{-y_1} \quad , y_1>0$$

$$F(x)=\int_0^x f(u)\,du=\left[\int_0^x e^{-u}\cdot du\right] * \left(\frac{-1}{-1}\right)$$

$$=-1\cdot\int_0^x -e^{-u}\,du=-1\left[e^{-u}\right]_0^x=-\left[e^{-x}-1\right]$$

$$F(x)=1-e^{-x} \quad , \quad F_x(y_1)=1-e^{-y_1}$$

$$\Rightarrow g(y_1)=5\,(1-F_x(y_1))^{4}\cdot f_x(y_1)$$

$$=5\,(1-(1-e^{-y_1}))^{4}\cdot e^{-y_1}=\begin{cases} 5(e^{-y_1})^{5} & , y_1>0 \\ 0 & , o.w. \end{cases}$$