2020/11/19 محاضرة الاربعاء

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الموضوع :   التاريخ / /

The joint distributions for the order statistics

For any $i<j$, the joint OS distribution Between y_i and y_j is given as:

$$g(y_i, y_j) = \frac{n!}{(i-1)!\cdot(j-i-1)!\,(n-j)!}\cdot[F_x(y_i)]^{i-1}$$

$$\cdot[F_x(y_j)-F_x(y_i)]^{j-i-1}\cdot[1-F_x(y_j)]^{n-j}\cdot f_x(y_i)\cdot f_x(y_j)$$

$$a<y_i<y_j<b.$$

Ex) let $X_i \sim Exp(1), i=1,\dots,5$ be OS.
find $g(y_2, y_4)$?

Sol) $n=5$, $i=2$ and $j=4$ then we have to find

$$g(y_2, y_4) = \frac{5!}{(2-1)!\,(4-2-1)!\,(5-4)!}\cdot[F_x(y_2)]^{2-1}\cdot$$

$$[F_x(y_4)-F_x(y_2)]^{4-2-1}\cdot[1-F_x(y_4)]^{5-4}\cdot f_x(y_2)\cdot f_x(y_4)$$

$\Rightarrow$ ~~$g(y_2,y_4) = \frac{5!}{(1)!}\dots$~~

$\because x \sim Exp(1)$

$$f(x)=\begin{cases} e^{-x} & , x>0 \\ 0 & , o.w. \end{cases}$$

$$f_x(y_2) = e^{-y_2} \quad \text{and} \quad f_x(y_4) = e^{-y_4}$$

$$F(x) = \int_0^x f(u)\,du = \int_0^x e^{-u}\,du = 1 - e^{-x}$$

$$\Rightarrow F_x(y_2) = 1 - e^{-y_2} \quad , \quad F_x(y_4) = 1 - e^{-y_4}$$

$$\Rightarrow g(y_2, y_4) = 120\,[F_x(y_2)] \cdot [F_x(y_4) - F_x(y_2)] \cdot [1 - F_x(y_4)] \cdot f_x(y_2) \cdot f_x(y_4)$$

$$\Rightarrow g(y_2, y_4) = \begin{cases} 120\, e^{-2y_4} \cdot e^{-y_2} [1 - e^{-y_2}][e^{-y_2} - e^{-y_4}] & , 0 < y_2 < y_4 < \infty \\ 0 & , \text{o.w} \end{cases}$$

Ex$_2$) let $f(x) = \begin{cases} 1 & , 0<x<1 \\ 0 & , o.w \end{cases}$

and Y_i, $i = 1,2,3$ are o.s. find $g(y_1, y_3)$ and $g(y_3)$

Sol) we have $n=3$, $i=1$ and $j=3$ then we have to find

$$g(y_1, y_3) = \frac{3!}{(1-1)!\,(3-1-1)!\,(3-3)!} \cdot [F_x(y_1)]^{1-1}$$

$$\cdot [F_x(y_3) - F_x(y_1)]^{3-1-1} \cdot [1 - F_x(y_3)]^{3-3}$$

$$\cdot f_x(y_1) \cdot f_x(y_3).$$

$f(x) = 1$, $0<x<1$ so $f_x(y_1) = 1$ and $f_x(y_3) = 1$

$$F(x) = \int_0^x f(u)\,du = \int_0^x du = x \rightarrow F_x(y_1) = y_1$$

$$F_x(y_3) = y_3$$

$$\therefore g(y_1, y_3) = \begin{cases} 6[y_3 - y_1] & , 0<y_1<y_3<1 \\ 0 & , o.w. \end{cases}$$

$$g(y_3) = 3\,(F_x(y_3))^{3-1} \cdot f_x(y_3)$$

$$g(y_3) = \begin{cases} 3y_3^2 & , 0<y_3<1 \\ 0 & , o.w \end{cases}$$