3- Poisson distribution:

A random variable is said to have a Poisson distribution if the probability mass function (pmf) of x given by:

$\boldsymbol{f(x;\lambda)} = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad x = 0,1,\dots,\infty \quad ,\lambda > 0$, it is denoted by **x~Po(λ)**

The moments are :

1/the mean → $E(x) = \lambda$

2/the variance → $v(x) = \lambda$

3/the m.g.f. → $M_x(t) = e^{\lambda(e^t-1)}$

Notes:

1-if the m.g.f. of x is exist ,then we can find mean and variance as:

$$M_x(t) = e^{\lambda(e^t-1)}$$

$$E(x) = M'_x(t)|_{t=0} \rightarrow \lambda e^t e^{\lambda(e^t-1)}|_{t=0} , E(x) = \lambda$$

$$E(x^2) = M''_x(t)|_{t=0} \rightarrow (\lambda e^t)^2 e^{\lambda(e^t-1)} + \lambda e^t e^{\lambda(e^t-1)}|t = 0$$

$$= \lambda^2 + \lambda$$

$$v(x) = E(x^2) - (E(x))^2$$

$$= \lambda^2 + \lambda - \lambda^2 = \lambda$$

2-For Binomial distribution if (n) is very large and (p) is very small , then the r.v. x gone to have Poisson distribution with parameter $\lambda = np$.

Ex.1 Let X be a r.v. with m.g.f. given by

$M_X(t) = e^{2(e^t - 1)}$, Find $P(X=3)$ and $P(X>2)$?

Sol.

من الداله المولده للعزوم المعطى من السؤال نعرف ماهو توزيع X لايجاد المطاليب الباقية.

$\Rightarrow X \sim Po(\lambda = 2)$ with pmf as:

$$f(x;\lambda) = \frac{e^{-2}\, 2^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

$$P(X=3) = f(3) = \frac{e^{-2}\, 2^{3}}{3!} = \boxed{0.18}$$

$$P(X>2) = 1 - P(X \leq 2)$$

هنا اخذنا المكملة للاحتمال

لان المطلوب $P(X>2)$ بمعناه يجب ايجاد القيم الى ∞

لان قيم المتغير X في هذا التوزيع الى ∞ فتكون

$$1 - P(X \leq 2) = 1 - [f(0) + f(1) + f(2)]$$

$$= 1 - \left[\frac{e^{-2}\, 2^{0}}{0!} + \frac{e^{-2}\, 2^{1}}{1!} + \frac{e^{-2}\, 2^{2}}{2!}\right] = \boxed{0.32}$$