Limits الغايات

هو مفهوم يستخدم لدراسة سلوك الدوال أو المتغيرات عندما تقترب من قيمة معينة، سواء كانت قيمة حقيقية او قيمة لا نهائية.

Definition:

A limit describes the value that a function approaches as the input (independent variable), For example, if we have a function $f(x)$, we are interested in the value that $f(x)$ gets closer to as x approaches a certain value a.

This is written as:

$$\lim_{x \to a} f(x) = L$$

This means "as x approaches a, $f(x)$ approaches the value L.

وهذا يعني أن "عندما يقترب x من a، تقترب $f(x)$ من القيمة L "

Types of Limits:

- Finite Limit: A limit is finite if the function approaches a specific, finite value as x approaches a.

$$\lim_{x \to a} f(x) = L$$

For example,

Consider the function $f(x) = 3x + 1$. find the limits of this function when $x \to 2$.

$$\lim_{x \to 2}(3x + 1)$$

To find this limit,

we simply substitute x =2 into the function

$$f(2) = 3(2) + 1 = 7$$

Thus,

$$\lim_{x \to 2}(3x + 1) = 7$$

- Infinite Limit**:** A limit is infinite if the function approaches infinity (either positive or negative) as x approaches a.

$$\lim_{x \to a} f(x) = \infty \ , \qquad \lim_{x \to a} f(x) = -\infty$$

For example,

Consider the function$f(x) = \frac{1}{x}$, Let's find the limit as $x \to 0^+$:

$$\lim_{x \to 0^+} \frac{1}{x}$$

Therefore,

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty$$

- Limit at Infinity: A limit at infinity happens when x approaches infinity (positive or negative).

$$\lim_{x\to \infty^+} f(x) \qquad , \qquad \lim_{x\to \infty^-} f(x)$$

For example,

Consider the function $f(x) = \frac{1}{x}$, Let's find the limit as

$x \to \infty$

$$\lim_{x\to \infty} \frac{1}{x}$$

As x increases without bound

Therefore,

$$\lim_{x\to \infty} \frac{1}{x} = 0$$

عندما تقترب x من ∞ بالاتجاه الموجب فأن $f(x) = \frac{1}{x}$ سوف تقترب من الصفر لان المقام

ونفس الحالة عندما تقترب x من ∞ من الاتجاه السالب. وعليه في كلا الحالتين تقترب الدالة من الصفر

Example: find

$$\lim_{x\to\infty} \frac{1}{x^3}$$

Solution/

$$\lim_{x\to\infty} \frac{1}{x^3} = \lim_{x\to\infty} (\frac{1}{x} . \frac{1}{x} . \frac{1}{x})$$

$$=(\lim_{x\to\infty} \frac{1}{x}).(\lim_{x\to\infty} \frac{1}{x}).(\lim_{x\to\infty} \frac{1}{x})$$

=0.0.0=0

∴

$$\lim_{x\to\infty} \frac{1}{x^3} = 0$$

Example: find

$$\lim_{x\to\infty} \frac{2x^3+1}{5x^3+x^2-1}$$

نقسم البسط والمقام على المتغير الذي يحمل اعلى اس

في هذا المثال اعلى اس هو x^3

Solution/

$$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{1}{x^3}}{5\frac{x^3}{x^3}+\frac{x^2}{x^3}-\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{1}{x^3}}{5+\frac{1}{x}-\frac{1}{x^3}}$$

$$= \frac{\lim\limits_{x\to\infty}(2) + \lim\limits_{x\to\infty}(\frac{1}{x^3})}{\lim\limits_{x\to\infty}(5) + \lim\limits_{x\to\infty}(\frac{1}{x}) - \lim\limits_{x\to\infty}(\frac{1}{x^3})}$$

$$\frac{\lim\limits_{x\to\infty}(2)+0}{\lim\limits_{x\to\infty}(5)+0-0} = \frac{2}{5}$$

Example:

Consider the function $f(x) = \frac{x^2-4}{x-2}$. Let's find the limit of this function as $x \to 2$.

Solution/

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = \lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)}$$

$$\lim_{x\to 2}(x+2) = 4$$

Example:

$$\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)$$

Solution:

$$\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)=\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}\right)$$

$$=\frac{1}{(\sqrt{x}+1)}=\frac{1}{2}$$

Homework: find

1. $\lim_{x\to-\infty}\frac{q_x^5-x^4+x^2-2}{4x^5+x^3-x+3}$
2. $\lim_{x\to\infty}\frac{x^4+x^2-x+1}{x^5+x^3-2x+3}$
3. $\lim_{x\to 10}(x^2+x-2)$
4. $\lim_{x\to 2}\left[5(2x-4)(x-1)^2\right]$
5. $\lim_{t\to 3}\left(\frac{t^3-1}{t-1}\right)$
6. $\lim_{x\to-\infty}\left(\frac{1}{x}\right)$

7. $\lim_{x\to 5}\left(\frac{x^2-25}{x-5}\right)$

8. $\lim_{x\to 0}\left(\frac{\sqrt{x+5}-\sqrt{5}}{x}\right)$

9. $\lim_{x\to 2}\left(\frac{\sqrt{x^2-x}}{x\sqrt{x-1}}\right)$

10. $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$

11. $\lim_{x\to -1}(1+2^x)$