Continuity الاستمرارية

Definition:

A function f is said to continuous at a if and only if the following conditions are satisfying:

1-$f(a)$ exists

2-$\lim_{x \to a} f(x)$ exists

3-$\lim_{x \to a} f(x) = f(a)$

يمكن ان نعرف الاستمرارية لدالة معينة هندسياً (بأن الدالة f مستمرة اذا لا يوجد قطع في بيان الدالة)

Example:

Let $f(x) = \begin{bmatrix} x^2-2 & ,if \quad x \geq 2 \\ x & ,if \quad x < 2 \end{bmatrix}$, is $f(x)$ continuous at $x = 2$

Solution:

1. $f(a)$ exists

To find $f(2) = (2)^2 - 2 = 2$

2-$\lim_{x \to a} f(x)$ exists

Right side

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - 2)$$

$$= 4 - 2 = 2 = L_1$$

Left side

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x)$$

$$= 2 = L_2$$

Thus $\lim_{x\to 2} f(x) = 2$

3- since $\lim_{x\to 2} f(x) = f(a) = 2$

Thus, f is continuous at $x = 2$

Example

$f(x) = |x|$, is $f(x)$ is cont. at $x = 0$

Solution:

$$|x| = \begin{bmatrix} x & ,x\geq 0 \\ -x & ,x<0 \end{bmatrix}$$

1. $f(a)$ exists, $f(0) = 0$
2. -$\lim_{x\to a} f(x)$ exists

Right side

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x) = 0$$

Left side

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$$

$$\lim_{x \to 0} |x| = 0$$

3- since $\lim_{x \to 0} f(x) = f(0) = 0$

Thus, f is continuous at $x = 0$

Example

Determine whether $f(x) = \begin{bmatrix} \frac{x^2-25}{x-5} & , \ if \ x \neq 5 \\ 8 & , \ if \ x = 5 \end{bmatrix}$ is continuous at $x = 5$?

Solution:

1. $f(a)$ exists, $f(5) = 8$
2. $\lim_{x \to a} f(x)$ exists

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$

$$= \lim_{x \to 5} \frac{(x-5)(x+5)}{(x-5)}$$

$$= \lim_{x \to 5} (x+5) = 10$$

3. $\lim_{x \to 5} f(x) \neq f(5)$, therefore $f(x)$ is not cont. at $x = 5$

Example

Determine whether $f(x) = \begin{bmatrix} 2^x & , \; if \; x \neq -1 \\ 3 & , \; if \; x = -1 \end{bmatrix}$ is continuous at $x = -1$?

Solution:

1- $f(a)$ exists, $f(-1) = 3$

3. 2- $\lim_{x \to a} f(x)$ exists

Right side

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2^x) = 2^{-1} = \frac{1}{2}$$

Left side

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (2^x) = 2^{-1} = \frac{1}{2}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = \frac{1}{2}$$

3- $f(-1) \neq \lim_{x \to -1} f(x) =$, therefore $f(x)$ is not cont.

at $x = -1$

Example:

Let $f(x) = \begin{bmatrix} 4\text{-}x & ,if \quad x<0 \\ x^2 & ,if \quad x\geq 0 \end{bmatrix}$, is $f(x)$ continuous at $x = 4$

Solution:

1. $f(a)$ exists

To find $f(4) = (4)^2 = 16$

2- $\lim_{x \to a} f(x)$ exists

$$\lim_{x \to 4} f(x) = \lim_{x \to 4}(x^2) = 4^2 = 16$$

$\therefore$ $f(4) = \lim_{x \to 4} f(x)$

Thus, f is continuous at $x = 4$

Homework

1- Let $f(x) = \frac{x^2-25}{x+5}$, find $\lim_{x \to -5} f(x)$

2- what value showed be assigned to b to make the function

$h(x) = \begin{bmatrix} x^3 & ,x<\frac{1}{2} \\ b\,x^2 & ,x\geq\frac{1}{2} \end{bmatrix}$ is continuous at $x = \frac{1}{2}$

3- is the function $f(x) = \begin{bmatrix} x & ,0\leq x\leq 3 \\ 4-x & ,3<x\leq 5 \end{bmatrix}$ cont.

1- at $x = 0$, 2- at $x = 2$ 3- at $x = 3$ 4- at $x = 5$

4- let $f(x) = \begin{bmatrix} k & ,if\ x=2 \\ \frac{x^2-4}{x-2} & ,if\ x\neq 2 \end{bmatrix}$,

Find k , such that f is cont. at $x = 2$

5- let $f(x) = \begin{bmatrix} \frac{\sin 4x}{3x} & ,if\ x\neq 0 \\ h & ,if\ x=0 \end{bmatrix}$,

if f is cont. at $x = 0$, find h