∴ Consistency ∴

Def:- An estimator $\hat{\theta}$ is called a consistent estimator for θ iff

1) $E(\hat{\theta}) = \theta$, $\hat{\theta}$ is unbiased estimator for θ
2) $\lim_{n \to \infty} Var(\hat{\theta}) = 0$.

Ex 1) let $x_i \sim N(\theta, 1)$, $i = 1, 2, \dots, n$. is $\hat{\theta} = \bar{x}$ a consistent estimator for θ?

Sol:-

$$\because x \sim N(\theta, 1) \quad \begin{cases} \to E(\theta) = ? \\ \to V(\theta) = ? \end{cases}$$

1-

$$\Rightarrow E(\hat{\theta}) = E(\bar{x}) = E\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \left(\frac{\sum_{i=1}^{n} E(x_i)}{n}\right)$$

$$= \frac{n\theta}{n} = \theta.$$

$\rightarrow$ So $\hat{\theta} = \bar{x}$ is unbiased estimator for θ.

2-

$$\lim_{n \to \infty} Var(\hat{\theta}) = \lim_{n \to \infty} Var(\bar{x}) = \lim_{n \to \infty} \left(\frac{1}{n}\right) = 0.$$

قوانين limit

$\therefore \hat{\theta} = \bar{x}$ is a consistent estimator for θ.

$\rightarrow x \sim N(\mu, \sigma^2) \rightarrow V(x) = \sigma^2$

$\because x \sim N(\theta, 1) \Rightarrow V(x) = 1$

* **Remark:-** if $x \sim N(\theta, \frac{1}{n})$, $i = 1, 2, \dots, n$, $\hat{\theta} = \bar{x}$ a consistent estimator for θ?

Ex2:- let $X_i \sim Bin(1,\theta)$, $i=1,2,\dots,n$. is $\hat{\theta}=\bar{x}$ a consistent estimator for θ?

solution:- 1. $E(\hat{\theta}) = E(\bar{x}) = E\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{n\theta}{n} = \theta$

$\therefore \hat{\theta}=\bar{x}$ is unbiased estimator for θ?

2- $\lim_{n\to\infty} Var(\hat{\theta}) = \lim_{n\to\infty} (Var(\bar{x})) = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} V(x_i)}{n^2}$

$= \lim_{n\to\infty} \frac{\sum_{i=1}^{n} \theta(1-\theta)}{n^2} = \lim_{n\to\infty} \frac{\theta(1-\theta)}{n} = 0.$

$\therefore \hat{\theta}=\bar{x}$ is a consistent estimator for θ.

Ex3:- let $X_i \sim Exp(\theta)$, $i=1,2,\dots,n$. is $\hat{\theta}=\bar{x}$ a consistent estimator for θ?

sol:- 1- $E(\hat{\theta}) = E(\bar{x}) = \sum_{i=1}^{n} \frac{E(x_i)}{n} = \frac{n\theta}{n} = \theta$

So $\hat{\theta}=\bar{x}$ is unbiased estimator for θ.

2- $\lim_{n\to\infty} Var(\hat{\theta}) = \lim_{n\to\infty} Var(\bar{x}) = \lim_{n\to\infty} \frac{\theta^2}{n} = 0.$

$\therefore \hat{\theta}=\bar{x}$ is a consistent estimator for θ.

H.W:- let $X_i \sim P(\theta)$, $i=1,2,\dots,n$. is $\hat{\theta}=\bar{x}$ a consistent estimator for θ?