∴ Uniqueness ∴

3- To find unbiased estimator?

$$T = \sum_{i=1}^{n} x_i \rightarrow \text{from Point } (1)$$

$\rightarrow$ we have $\hat{\theta} = \frac{T}{n} = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{X}$

$$\rightarrow E(\hat{\theta}) = E(\bar{x}) = \frac{\sum_{i=1}^{n} E(x_i)}{n} = \frac{\cancel{n}\theta}{\cancel{n}}$$

$\therefore \hat{\theta} = \bar{x}$ is the best Unique estimator for θ.

H.W: let $X_i \sim Ber(\theta)$, $i = 1, 2, \dots, n$
is $\bar{x}$ unique estimator for θ?

∴ Uniqueness ∴

To find the unique estimator for the parameter θ, we found the following steps:-

1- find $T(x)$ the comp. suff for θ.
2- find the expected value of T.
3- find the unbiased estimator $\hat{\theta}$ as a function of T.

⇒ Then $\hat{\theta}$ is the best unique estimator for θ.

Ex1:- let $X_i \sim Exp(\theta)$, $i=1,2,\dots,n$. is $\bar{X}$ unique estimator for θ?

Solution:- ① find $T(x)$ the comp S.S for θ.

$\because x \sim Exp(\theta)$

$$\Rightarrow f(x,\theta) = \exp[\ln f(x,\theta)] = \exp\left[\ln\left(\frac{1}{\theta}\cdot e^{-\frac{x}{\theta}}\right)\right]$$

$$= \exp\left[\ln\left(\frac{1}{\theta}\right) - \frac{x}{\theta}\right] = \exp\left[\ln(1) - \ln(\theta) - \frac{x}{\theta}\right]$$

$a(\theta) = \ln(1) - \ln(\theta)$, $b(x) = 0$, $c(\theta) = \frac{-1}{\theta}$

$d(x) = x$ $\Rightarrow T = \sum_{i=1}^{n} d(x_i) = \sum_{i=1}^{n} x_i$ is a Comp S.S. for θ.

2- Find the expected value of T.

$$\Rightarrow E(T) = E\left(\sum_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} E(x_i) = \sum_{i=1}^{n} \theta = n\theta.$$

3- To find unbiased estimator for T?

from Point(1) $\rightarrow T = \sum_{i=1}^{n} x_i$