Limits

Remark :-

1- if the value of $f(x)$ approaches the number L_1 as x approaches to x_0 from the right side.

we write $\lim\limits_{x \to x_0^+} f(x) = L_1$ (Right hand Limit).

2- if the value of $f(x)$ approaches the number L_2 as x approaches to x_0 from the left side

we write $\lim\limits_{x \to x_0^-} f(x) = L_2$ (left hand limit)

i,e $\lim\limits_{x \to x_0} f(x) \begin{cases} \lim\limits_{x \to x_0^+} f(x) = L_1 \\ \lim\limits_{x \to x_0^-} f(x) = L_2 \end{cases}$

* if the limit from the left side is the same as the limit from the right i,e if $L_1 = L_2 = a$

$\Rightarrow \lim\limits_{x \to x_0} f(x) = a$ exist.

but if $L_1 \neq L_2 \rightarrow \lim\limits_{x \to x_0} f(x)$ is not exist.

Def:- when a function $f(x)$ tends to the number L as x tends to the number a, we write $f(x) \rightarrow L$ as $x \rightarrow a$ or $\lim\limits_{x \to a} f(x) = L$.

∴ Limits ∴

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Ex:- Find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2} f(x)$, if it exist?

$$f(x)=\begin{cases} 1, & x\geq 2 \\ -1, & x<2 \end{cases}$$

$\boxed{\text{Sol}}$:- $\underbrace{\lim_{x\to 2^+} f(x)=1}_{=L_1}$, $\underbrace{\lim_{x\to 2^-} f(x)=-1}_{=L_2}$

$$\therefore L_1=\lim_{x\to 2^+} f(x) \neq L_2=\lim_{x\to 2^-} f(x)$$

$\therefore \lim_{x\to 2} f(x)$ is not exist.

Ex②:- Find $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a} f(x)$

if it exist? if $a=1$, $a=2$

$$f(x)=\begin{cases} x^2+1, & x\leq 1 \\ x+1, & 1<x<2 \\ 3, & x\geq 2 \end{cases}$$

$\boxed{\text{Sol}}$:- Case 1:- if $a=2$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} 3=3=L_1$$

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-} (x+1)=3=L_2$$

Since $L_1=L_2 \rightarrow \lim_{x\to 2} f(x)=3$

$\therefore \lim f(x)$ is exist.

Case 2:-

if $a=1$

$$L_1 = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x+1) = 2$$

$$L_2 = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2+1) = 2$$

since $L_1 = L_2 \rightarrow \lim_{x \to 1} f(x) = 2$

$\therefore \lim_{x \to 1} f(x)$ is exist.

∴ Theorems of limits ∴

Theorem 1:- if f and g are two functions with $\lim\limits_{x\to a} f(x) = \mathcal{L}_1$

$\lim\limits_{x\to a} g(x) = \mathcal{L}_2$, i,e the limits exist

The two functions have limits.

1- $\lim\limits_{x\to a} c = c$ (c is any constant).

2- $\lim\limits_{x\to a} x = a$, 3- $\lim\limits_{x\to a} c\,f(x) = c\mathcal{L}_1$

4- $\lim\limits_{x\to a} (f(x) \pm g(x)) = \lim\limits_{x\to a} f(x) \mp \lim\limits_{x\to a} g(x)$

$= \mathcal{L}_1 \mp \mathcal{L}_2$ (limit of Sum and difference).

5- $\lim\limits_{x\to a} (f(x) . g(x)) = \lim\limits_{x\to a} f(x) . \lim\limits_{x\to a} g(x)$

$= \mathcal{L}_1 . \mathcal{L}_2$ (limit of Product).

6- $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \dfrac{\mathcal{L}_1}{\mathcal{L}_2}$, Provided $\mathcal{L}_2 \neq 0$ (limit of a quotient).

Ex:- Find lim of the following functions:-

1- $\lim\limits_{x\to 2} 2x = 4$

2- $\lim\limits_{x\to -2} 10 = 10$

3- $\lim\limits_{x\to 0} (x^2 - 2x + 1) = 1$