Ordinary differential equation

Definition 1: An equation expressing a relation between a function and its derivative and the variables is called a differential equation (D.E.).

* Differential equations are classified into

1- Ordinary differential equation (ODE)

2- partial differential equation (PDE)

Definition 2:a differential equation (DE) containing a derivative function of a single variable is called an Ordinary differential equation.

Definition 3: A differential equation (DE) containing a partial derivative function of more than one variable, with each derivative relation to one of the variables, is called a partial differential equation.

Examples:

1- $\frac{d^2y}{dx^2} = 4y$

2- $yy'' - (y')^3 = lnx$

3- $s = t\frac{ds}{dt} + \sqrt{1 - \left(\frac{ds}{dt}\right)^3}$

4- $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 1$

5- $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial r} = u$

Definition 4:the order of (DE) is the order of the highest derivative appearing in it.

*The general form of an nth order Ordinary differential equation in variables (x) and (y) is

$f\left(x, y', y'', \dots\dots\dots\dots, y^{(n)}\right) = 0$ where $y^{(n)} = \frac{d^n x}{dx^n}$

Definition 5: The degree of a differential equation is the degree of the power of highest derivative.

Definition 6: The function $y = y(x)$ is called a solution to an ODE on the open interval I if it satisfies the equation and is defined on I.

Definition 7: the problem of an ODE with the initial condition $y(x_0) = y_0$ is called an initial value problem (IVP).

Definition 8: The problem of an ODE with the boundary condition is called a boundary value problem (BVP).

1- $y'' + y = 0$ $\quad y(0) = 1 \quad y'(0) = 2$ is IVP
2- $y' + x^2 = y''$ $\quad y(0) = 1 \quad y'(0) = 2 \quad y''(1) = 2$ is BVP

Definition 9:

a) The set of all solutions of (DE) is called the general solution.
b) Any special solution of (DE) that is mainly obtained under certain conditions is called a particular solution.
c) A differential equation may sometimes have an additional solution that cannot be obtained from the general solution is called a singular solution.

Examples:

1- $y' = \cos x$ where $y = \sin x + c$ is the general solution
2- $y' = \cos x$ $\quad y(0) = 1$ where $y = \sin x + 1$ is the particular solution
3- $(y')^2 - xy' + y = 0$, the general solution is $y = cx - c^2$ and $y = \frac{x^2}{4}$ is a singular solution

Definition 10: ODE is called linear if the function (dependent variable) and derivatives are linear.

Definition 11: ODE is called homogenous if equation equal zero.

Definition 12: a system of ODE is a simullaneous set of equation that involves two or more dependent variables that dependent on one independent variable.

Example for each form equation:

1- $y = \sin x + x$
2- $y_t = (y_x)^4 + e^{t+x}$
3- $y'' + y = \tan x$
4- $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = y^2 lny$
5- $\frac{d^3x}{dy^3} - 9y = 0$
6- $4xy\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3 = 0$
7- $x\frac{\partial^2 u}{\partial x^2} + y\left(\frac{\partial^2 u}{\partial y^2}\right) = 1$
8- $2y + x^2y'' = 2xy'$
9- $y = xy' + \frac{2}{y'}$
10- $xyy' + x(y')^2 - yy' = 0$