

Composition of the differential equation form the solution

EX. find the differential equation form the solution

$$1- 3x + c = 2y^2$$

$$3 = 4y \frac{dy}{dx}$$

$$y \frac{dy}{dx} = \frac{3}{4} \quad \text{is diff.eq.}$$

$$2- cx^2 = y + 5 \longrightarrow c = \frac{y+5}{x^2}$$

$$2cx = \frac{dy}{dx}$$

$$2 \left(\frac{y+5}{x^2} \right) = \frac{dy}{dx} \quad \text{is diff.eq.}$$

$$3- c_1x = c_2x^2 - y$$

$$c_1 = 2c_2x - y'$$

$$0 = 2c_2 - y'' \longrightarrow c_2 = \frac{y''}{2}$$

$$c_1 = 2 \left(\frac{y''}{2} \right) x - y' \longrightarrow c_1 = y''x - y'$$

$$(y''x - y')x = \left(\frac{y''}{2} \right) x^2 - y$$

$$x^2y'' - xy' = \frac{1}{2}x^2y'' - y$$

$$\frac{1}{2}x^2 - xy' + y = 0 \quad \text{is diff.eq.}$$

$$4- xy = ae^x + be^{-x} + x^2 \dots\dots\dots(1)$$

$$xy' + y = ae^x - be^{-x} + 2x \dots\dots\dots(2)$$

$$xy'' + y' + y' = ae^x + be^{-x} + 2$$

$$xy'' + 2y' = ae^x + be^{-x} + 2 \dots\dots\dots(3)$$

From (1) and (3) we get

$$xy'' + 2y' - xy = 2 - x^2$$

H.W. find the differential equation form the solution

a) $y = ce^{2x}$

b) $2xy^2 = c - 5$

c) $y = c_1e^{2x} + c_2e^{-2x}$

d) $y = c_1 \cos 2x + c_2 \sin 2x$

e) $y = ax^2 + bx + c$

f) $\sec(x + y) = x + c$