First order first degree ODE

The simplest type of ODE is that of the first order and first degree is of the form.

1- $f(x, y, y')$

2- $y' = f(x, y)$ or $\frac{dy}{dx} = f(x, y)$

3- $M(x, y)dx + N(x, y)dy = 0$

Since (1), (2) and (3) are equivalent

Examples:

1- $3yy' + 4x^2 - 2x = 0$ $\longrightarrow$ $f(x, y, y')$

2- $y' = \frac{2x-4x^2}{3y}$ $\longrightarrow$ $y' = f(x, y)$

3- $3ydy + (4x^2 - 2x)dx) = 0$ $\longrightarrow$ $M(x, y)dx + N(x, y)dy = 0$

1. Solve the ODE of First order, first degree

1.1. Variable separable equation: In this case, the ODE can be written in the form $M(x, y)dx + N(x, y)dy = 0$ integration of both sides gives the general solution

$$\int M(x, y)dx + \int N(x, y)dy = c$$

Examples: find the general solution of the ODE

1- $x(y^2 - 1)dx - y(x^2 - 1)dy = 0$

$[x(y^2 - 1)dx = y(x^2 - 1)dy]$ $\div (y^2 - 1)(x^2 - 1)$

$\int \frac{x}{x^2-1} dx = \int \frac{y}{y^2-1} dy$

$\frac{1}{2} ln|x^2 - 1 =| \frac{1}{2} ln|x^2 - 1| + c$

2- $\frac{dy}{dx} = \frac{(2lnx+1)}{\sin y + y\cos y}$

$$\int (\sin y + y\cos y)dy = \int (2lnx + 1)dx$$

$$-\cos y + y\sin y + \cos y = 2(xlnx - x) + x$$

$$y\sin y = 2xlnx - x + c$$

3- $3e^x(1 + \tan y)dx + (1 - e^x)\sec^2 y\, dy = 0$

$$[(1 - e^x)\sec^2 y\, dy = -3e^x(1 + \tan y)dx] \ \div (1 - e^x)(1 + \tan y)$$

$$\int \frac{\sec^2 y}{1 + \tan y} dy = \int \frac{-3e^x}{1 - e^x} dx$$

$$ln|1 + \tan y| = 3ln|1 - e^x| + c$$

4- $\sqrt{x^2 + x^2y^2}dx + \sqrt{y^2 - x^2y^2}dy = 0$

$$x\sqrt{1 + y^2}dx + y\sqrt{1 - x^2}dy = 0$$

$$\left[x\sqrt{1 + y^2}dx = -y\sqrt{1 - x^2}dy\right] \ \div (\sqrt{1 + y^2})(\sqrt{1 - x^2})$$

$$\int \frac{x}{\sqrt{1 - x^2}} dx = \int \frac{-y}{\sqrt{1 + y^2}} dy$$

$$\int x(1 - x^2)^{\frac{-1}{2}} dx = \int -y(1 + y^2)^{\frac{-1}{2}} dy$$

$$-\frac{1}{2}\int (-2x)(1 - x^2)^{\frac{-1}{2}} dx = -\frac{1}{2}\int (2y)(1 + y^2)^{\frac{-1}{2}} dy$$

$$(1 - x^2)^{\frac{1}{2}} = (1 + y^2)^{\frac{1}{2}} + c$$

$$\sqrt{1 - x^2} = \sqrt{1 - y^2} + c$$

5- $\frac{dy}{dx} = (x + y + 2)^2$

Let $u = x + y + 2 \longrightarrow \frac{du}{dx} = 1 + \frac{dy}{dx} \longrightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = u^2 \longrightarrow \frac{du}{dx} = u^2 + 1$

$\int \frac{1}{u^2+1} du = \int dx \longrightarrow \tan^{-1} u = x + c$

$\tan^{-1}(x + y + 2) = x + c$

6- $(x - y)^2 \frac{dy}{dx} = 9$

Let $u = x - y \longrightarrow \frac{du}{dx} = 1 - \frac{dy}{dx} \longrightarrow \frac{dy}{dx} = 1 - \frac{du}{dx}$

$u^2 \left(1 - \frac{du}{dx}\right) = 9 \longrightarrow u^2 - u^2 \frac{du}{dx} = 9$

$u^2 \frac{du}{dx} = u^2 - 9 \longrightarrow \frac{du}{dx} = \frac{u^2-9}{u^2}$

$\frac{u^2}{u^2-9} du = dx \longrightarrow \frac{u^2-9+9}{u^2-9} du = dx$

$\int 1 + \frac{9}{u^2-9} du = \int dx \longrightarrow u + \frac{3}{2} ln \left|\frac{u-3}{u+3}\right| = x + c$

$x - y + \frac{3}{2} ln \left|\frac{x - y - 3}{x - y + 3}\right| = x + c$

H.W. Find the general solution of the differential equation

a) $y' = 2xy^2 + y^2$ b) $\sin x \sin y + y' \cos y = 0$

c) $\frac{2x}{1-y^2} = \frac{dx}{dy}$ d) $xy' + \left(1 + \frac{1}{lnx}\right) y = 0$

e) $x \frac{dy}{dx} + (\sqrt{1 - y^2}) \cos^{-1} y = 0$ f) $\frac{dy}{dx} = \sin x + y$

g) $(3y - 1)e^{-2y^2} \frac{dy}{dx} = 4xe^{x^2-2y^2}$ h) $xydy - \frac{1+y^2}{1+x^2} dx = 0$

i) $(3xy^2 - 6x - y^2 + 2)dy - 3xy^2 dx = 0$

j) $y - x \frac{dy}{dx} = 3(1 + x^2 \frac{dy}{dx})$