System of simultaneous linear equation

Examples: solve the simultaneous equation

1- $\frac{ds}{dt} + s = e^t$ ………...(1)

$\frac{dz}{dt} = s$ ……………….…….…(2)

Take (1)

$\frac{ds}{dt} + s = e^t$ ………..(1) $P((t) = 1$ $Q(t) = e^t$

$I(t) = e^{\int P(t)dt} \rightarrow I(t) = e^{\int 1\ dt} \rightarrow I(t) = e^t$

$I(t).s = \int I(t).Q(t)\ dt \rightarrow e^t.s = \int e^t.e^t dt$

$e^t.s = \int e^{2t} dt \rightarrow e^t.s = \frac{1}{2}e^{2t} + c$

$s = \frac{1}{2}e^t + ce^{-t}$ …………………(3)

$\frac{dz}{dt} = \frac{1}{2}e^t + ce^{-t} \rightarrow dz = \frac{1}{2}e^t + ce^{-t}\ dt$

$\int dz = \int \frac{1}{2}e^t + ce^{-t}\ dt$

$z = \frac{1}{2}e^t - ce^{-t} + c1$

2- $rdx - (3r - 2x)dr = 0$ ………..(1)

$2r(2rx + 1)dr = dy$ ……………(2)

Take (1)

$rdx - (3r - 2x)dr = 0 \rightarrow rdx = (3r - 2x)dr$

$r\frac{dx}{dr} + 2x = 3r \rightarrow \frac{dx}{dr} + \frac{2}{r}x = 3$ $P(r) = \frac{2}{r}$ $Q(r) = 3$

$$I(r) = e^{\int P(r)dr} \;\rightarrow\; I(r) = e^{\int \frac{2}{r}\,dr} \;\rightarrow\; I(r) = e^{2\ln(r)} = r^2$$

$$I(r).x = \int I(r).Q(r)\,dr \;\rightarrow\; r^2.x = \int r^2.3\,dr$$

$$r^2.x = r^3 + c \;\rightarrow\; x = r + cr^{-2} \;\ldots\ldots\ldots\ldots(3)$$

$$2r[2(r + cr^{-2})r + 1]dr = dy$$

$$2r[(2r^2 + 2cr^{-1}) + 1]dr = dy$$

$$\int 4r^3 + 4c + 2r\;\,dr = \int dy$$

$$y = r^4 + 4rc + r^2 + c1$$

3- $\frac{dt}{2t} = \frac{dx}{x-t} = \frac{dy}{x+y}$

$$\frac{dt}{2t} = \frac{dx}{x-t} \ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{dt}{2t} = \frac{dy}{x+y} \ldots\ldots\ldots\ldots\ldots(2)$$

Take (1)

$$\frac{dt}{2t} = \frac{dx}{x-t} \;\rightarrow\; \frac{dx}{dt} = \frac{x-t}{2t} \;\rightarrow\; \frac{dx}{dt} = \frac{1}{2t}x - \frac{1}{2}$$

$$\frac{dx}{dt} - \frac{1}{2t}x = -\frac{1}{2} \qquad P(t) = \frac{-1}{2t} \quad Q(t) = \frac{-1}{2}$$

$$I(t) = e^{\int P(t)dt} \;\rightarrow\; I(t) = e^{\int \frac{-1}{2t}dt} \;\rightarrow\; I(t) = t^{\frac{-1}{2}}$$

$$I(t).x = \int I(t).Q(t)\,dt \;\rightarrow\; t^{\frac{-1}{2}}.x = \int t^{\frac{-1}{2}}.\frac{-1}{2}\,dt$$

$$t^{\frac{-1}{2}}.x = -t^{\frac{1}{2}} + c \;\rightarrow\; x = -t + ct^{\frac{1}{2}} \;\ldots\ldots\ldots\ldots(3)$$

$$\frac{dt}{2t}=\frac{dy}{-t+ct^{\frac{1}{2}}+y} \rightarrow \frac{dy}{dt}=\frac{-t+ct^{\frac{1}{2}}+y}{2t}$$

$$\frac{dy}{dt}=\frac{-1}{2}+\frac{c}{2}t^{\frac{-1}{2}}+\frac{y}{2t}$$

$$\frac{dy}{dt}-\frac{y}{2t}=\frac{-1}{2}+\frac{c}{2}t^{\frac{-1}{2}} \qquad P(t)=\frac{-1}{2t} \quad Q(t)=\frac{-1}{2}+\frac{c}{2}t^{\frac{-1}{2}}$$

$$I(t)=e^{\int P(t)dt} \rightarrow I(t)=e^{\int\frac{-1}{2t}dt} \rightarrow I(t)=t^{\frac{-1}{2}}$$

$$I(t).y=\int I(t).Q(t)\,dt \rightarrow t^{\frac{-1}{2}}.y=\int t^{\frac{-1}{2}}.(\frac{-1}{2}+\frac{c}{2}t^{\frac{-1}{2}})dt$$

$$t^{\frac{-1}{2}}.y=\int\frac{-1}{2}t^{\frac{-1}{2}}+\frac{c}{2t}\;dt \rightarrow t^{\frac{-1}{2}}.y=-t+\frac{c}{2}lnt+c1$$

$$y=t^{\frac{3}{2}}+\frac{c}{2}t^{\frac{1}{2}}lnt+c1\;t^{\frac{1}{2}}$$

H.W. solve the following simultaneous equation

1-- $\frac{dx}{dt}+y=x\ \dots\dots\dots(1)$

$\frac{dy}{dt}=3y\ \dots\dots\dots\dots(2)$

2-- $\frac{dy}{dt}+x^2=2t^2\ \dots\dots\dots\dots(1)$

$2tx\,dx+(t^2-x^2)dt=0\ \dots\dots(2)$

3-- $\frac{dx}{x}=\frac{dy}{x-y}=dt$