The derivative of the trigonometric functions

Theorem:

Let u be function then.

1- $d(sinu) = cosu.du$

2- $d(cosu) = -sinu.du$

3- $d(tanu) = sec^2u.du$

4-$d(cotu) = -\ csc^2u.du$

5- $d(secu) = secu\ .tanu\ .\ du$

6- $d(cscu) = -cscu\ .cotu\ .\ du$

Example:

Find $y' = \frac{dy}{dx}$ of the following functions.

1- $f(x) = x^3 + \sin 2x$

2- $y = \frac{\sin x}{(\tan x)+1}$

Sol/

1- $$y' = \frac{dy}{dx} = 3x^2 + [\cos 2x\ .(2)]$$

$$= 3x^2 + 2\cos 2x$$

2- $$y' = \frac{(tan\,x+1)[cos\,x\,.1]-\left[sin\,x\left(sec^2x+0\right)\right]}{(tan\,x+1)^2}$$

$$= \frac{\tan x \cos x + \cos x - \sin x\ sec^2x}{(\tan x + 1)^2}$$

$$\frac{\frac{\sin x}{\cos x}\cos x+\cos x-\sin x\, sec^2 x}{(\tan x+1)^2}$$

$$\frac{\sin x\ +\cos x-\sin x\, sec^2 x}{(\tan x+1)^2}$$

Home work:

Find $y' = \frac{dy}{dx}$ of the following functions.

1- $y = 4\sec x + \tan x$

2- $y = (\csc x).(x+1)$

3_ $y = \sqrt{x} + \cot x$

4_ $y = \frac{x^2+2}{\cos x^2}$

Example:

Find y' for each of the following functions.

1- $y = \sin(x^2 + 2x + 1)$

Sol/ $y' = \cos(x^2 + 2x + 1)\,[2x + 2 + 0]$

$$= (2x+2).\cos(x^2 + 2x + 1)$$

2- $y = \tan\,(x + \cos x)$

Sol/

Let $u = x + \cos x$

$$y = \tan u$$

$$y' = sec^2 u.du$$

Now, $u = x + \cos x$

$$du = 1 + (-\sin x\ (1))$$

$$= 1 - \sin x$$

$$\Rightarrow y' = sec^2(x + \cos x).(1 - \sin x)$$

$$= sec^2(x + \cos x) - \sin x\ .\, sec^2(x + \cos x)$$

<u>Home work:</u> Find y' for each of the following functions.

1- $y = \cot(\sec x)$

2- $y = \frac{\sin(4x)}{x^2 + \cos x}$

3- $y = \frac{\sqrt{x}}{\sin x}$

4- $y = \frac{\tan(\sin x)}{\cot(\cos x)}$

Derivative of composite functions

Chain Rule :

Let $y = f(u)$, $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ قاعدة السلسلة

Example:

لدينا دالتين الاولى هي y معرفة بدلالة u والثانية هي u معرفة بدلالة x والمطلوب هو مشتقة ال y بدلالة ال x وعليه نطبق قاعدة السلسلة

Let $y = 5u^2 + 3u$, $u = \sin x$ find $\frac{dy}{dx}$

Sol/

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Now $\frac{dy}{du} = 10u + 3$ and $\frac{du}{dx} = \cos x$

Thus $\frac{dy}{dx} = (10u + 3)(\cos x)$

نعوض بدل كل u بما يساويها لكي نحصل على مشتقة y بدلالة x

$$\Rightarrow \frac{dy}{dx} = (10(\sin x) + 3)(\cos x)$$

$$= 10 \sin x \cos x + 3 \cos x$$

Example:

Find $\frac{dy}{dx}$ for each of the following functions

1- $y = \sin(\tan x)$

Sol/

Let $u = \tan x \Rightarrow y = \sin u$

$$\frac{dy}{du} = \cos u \quad \text{and} \quad \frac{du}{dx} = sec^2 x$$

Thus by chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u \,.\, \sec^2 x = \cos(\tan x).\sec^2 x$$

2- $y = \left(x^2 + 5x\right)^{10}$

Sol/

Let $u = x^2 + 5x \Rightarrow y = u^{10}$

$$\frac{dy}{du} = 10 \,.\, u^9 \quad \text{and} \quad \frac{du}{dx} = 2x + 5$$

by chain rule

$$\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx} = \left(10 . u^9\right)(2x + 5)$$

$$= 10\left(x^2 + 5x\right)^9 (2x + 5)$$

3-$y = \cos(\sec x)$ **H.W**

Second Order Derivative and Derivatives of Higher Order

The derivative $\acute{y} = \frac{dy}{dx}$ is the first derivative of $y = f(x)$ with respect to x. The first derivative is itself a function of x and may be differentiable.

Thus, we can derivative it

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2 f}{d^2 x} = \frac{d^2 y}{d^2 x}$$

Which is called second derivative of $y = f(x)$ with respect to x .

In general

$$y''' = \frac{d^3y}{d^3x} = \frac{d}{dx}\left(\frac{d^2y}{d^2x}\right) \text{ is the third derivative.}$$

$$y'''' = y^{(4)} = \frac{d^4y}{d^4x} = \frac{d}{dx}\left(\frac{d^3y}{d^3x}\right) \text{ is the fourth derivative.}$$

$\vdots$

$\vdots$

$$y^{(n)} = \frac{d^ny}{d^nx} = \frac{d}{dx}\left(\frac{d^{n-1}y}{d^{n-1}x}\right) \text{ is n the derivative.}$$

Example:

Find $\grave{y}$ y'' y''' $y^{(4)}$ of $y = \sin(x^3)$

Sol/

$$\grave{y} = \cos(x^3).3\,x^2 = 3x^2\cos x^3$$

$$y'' = (3x^2)\left(-\sin x^3\,(3x^2)\right) + (\cos x^3)(6x)$$

$$= -9x^4\sin x^3 + 6x\ \cos x^3$$

$$y''' = \left[-9\,x^4\left(3x^2\cos(x^3)\right) + \left(\sin(x^3)\right)\left(-36x^3\right)\right]$$
$$+\ [6x\ \left(-\sin x^3\,(3x^2)\right) + 6\cos x^3]$$

$$y^{(4)} \qquad \rightarrow \qquad H.M$$

Example:

Let $y = x^4 + 3x^2 - 5x + 2$ find $y'''(\frac{1}{6})$ and $y^{(4)}(\frac{1}{6})$

Sol/

$$\grave{y} = 4x^3 + 6x - 5$$

$$y'' = 12x^2 + 6$$

$$y''' = 24x \qquad y'''\left(\frac{1}{6}\right) = (24)\left(\frac{1}{6}\right) = 4$$

$y^{(4)} = 24 \rightarrow \; y^{(4)}\left(\frac{1}{6}\right) = 24$

Home work:

1. let $f(x) = x \sin x$ find $f'\left(\frac{\pi}{4}\right)$
2. show that $y'' + 4y = 0$, when $y = 3\sin(2x + 3)$
3. show that $y''' + y'' + \grave{y} + y = 0$ when $y = \sin x + 2\cos x$
4. Find $\grave{y}$ y'' y''' for the following functions
   - I. $y = x^4 + 1$
   - II. $y = \cot x$
   - III. $y = 5\sec x$
   - IV. $y = \sqrt{\sin x}$
   - V. $y = \cos(x + 2)^2$
   - VI. $y = \frac{1}{2}\tan x \sin 2x$