



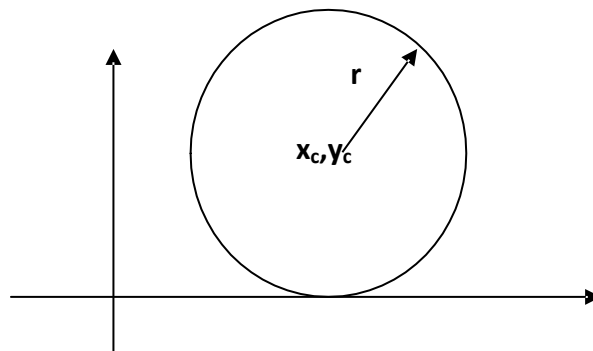
Computer Graphics

Forth Chapter

2025-2026

Circle Drawing

The circle is a special kind of curves. The circle is a closed curve with same starting and ending point. Circles are probably the most used curves in elementary graphics.



- A circle is specified by the coordinates of its center (x_c, y_c) and its radius (r) .
- The circle equation is : $(x-x_c)^2 + (y -y_c)^2 = r^2$ (1)
- If the center of the circle is at the origin $(0,0)$ then the equation is :

$$x^2+y^2=r^2 \text{ (2)}$$

Solving equation (1) for y :

$$y= y_c \pm \sqrt{r^2 - \sqrt{(x-x_c)^2}} \text{ (3)}$$

Note: To draw a circle increment the x values by one unit from $-r$ to $+r$ and use the above equation to solve for the two y values at each .step

1. Direct (implicit) algorithm

In this method the first pixel of circle is at left side as equation

$$x = x_c - r$$

$$y = y_c$$

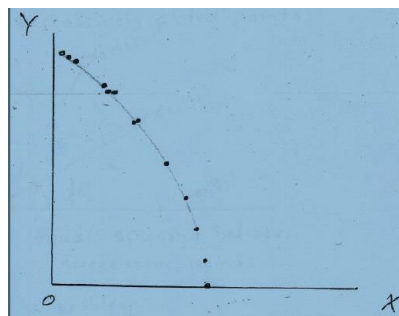
to draw the circle we can increment x from -r to +r or from 0 to 2r by one unit at each step and solving for y

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$

$$x = x + 1$$

This method of drawing a circle is inefficient because:

1. We are not taking advantages of the symmetry of the circle.
2. The amount of processing time required to perform the squaring and square root operations repeatedly.
3. X values are equally spaced (they differ by one unit) the y values are not. The circle is denser and flatter near the y-axis and has large gaps and is steep near the x-axis.



Direct Algorithm

```
Input : xc ,yc , r.  
Output : Circle  
{ x=xc-r;  
  for i= 0 to 2*r  
    { y=yc+ $\sqrt{r^2 - (x - xc)^2}$   
      plot (x, integer (y) ,color)  
      y=yc- $\sqrt{r^2 - (x - xc)^2}$   
      plot (x, integer (y),color)  
      x=x+1;  
    }  
  }
```

H\W: Design implicit algorithm to draw circle if the first point is at right side.

H\W: design implicit algorithm to draw circle if the first point is $x=xc, y=yc - r$

H\W: Find the point of a circle where $xc=20, yc = 10$ and $r=8$?

Example :Find the point of a circle where $x_c=10$, $y_c= 10$ and $r=5$ using direct algorithm?

$x_c=10$

$y_c=10$

$x=x_c-r$; $x=10-5=5$

For $i=0:2*r$

$y=y_c+\text{sqrt}((r^2)-(x-x_c)^2)$

Plot(x ,round(y),'y')

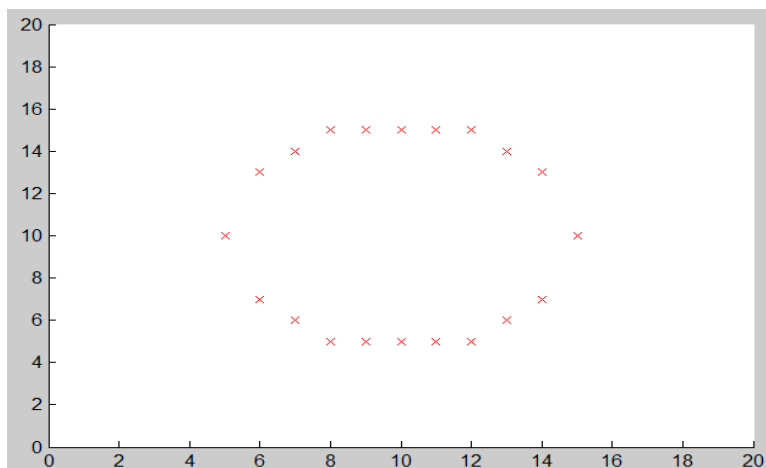
$y=y_c-\text{sqrt}((r^2)-(x-x_c)^2)$

Plot(x ,round(y),'y')

$x=x+1$

End

X	Y	Round(y)	Y	Round(y)	Plot(X,Y)
5	10	10	10	10	(5,10),(5,10)
6	13	13	7	7	(6,13),(6,7)
7	14	14	6	6	(7,14),(7,6)
8	14.5	15	5.4	5	(8,15),(8,5)
9	14.8	15	5.1	5	(9,15),(9,5)
10	15	15	5	5	(10,15),(10,5)
11	14.8	15	5.1	5	(11,15),(11,5)
12	14.5	15	5.4	5	(12,15),(12,5)
13	14	14	6	6	(13,14),(13,6)
14	13	13	7	7	(14,13),(14,7)
15	10	10	10	10	(15,10),(15,10)



2. parametric (polar) algorithm

One method of eliminating the problem of plotting points evenly spaced around the circle is to use polar representation of a circle:

$$x = x_c + r \cos \theta ,$$

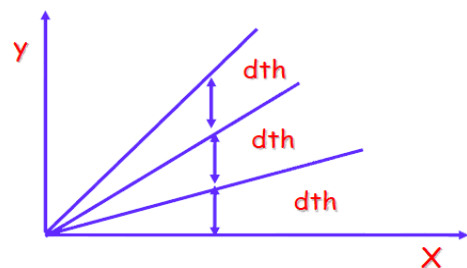
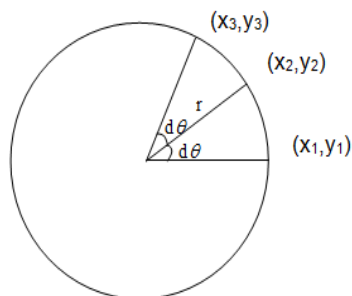
$$y = y_c + r \sin \theta .$$

Where: $\theta \rightarrow$ is measured in radians from 0 to 2π

arc length = $r \times \theta$, r =radius (constant)

in this method we depend on angles to draw the circle, since it propose the first angle $\theta=0$, and end angle is two_pi (360).

The change in angle ($d\theta$) must be small value $d\theta=1/r$.



Polar algorithm

```
Input : xc ,yc , r.  
Output : Circle  
{  
  th=0; dth=1/r;  
  while (th<=2*pi)  
  {  
    x = xc + r *cos (th)  
    y = yc + r sin(th)  
    plot (round(x),round(y),color)  
    th = th + dth  
  }  
}
```

Note: the algorithm use cos & sin operation and do not take the advantage of symmetric in circle

H\W: write Matlab program to draw circle using polar algorithm?

Example :Find the point of a circle where $x_c=10$, $y_c= 10$ and $r=5$ using polar algorithm ?

```

th=0
dth=1/r=1/5
While th <=2*pi
    x=xc+r*cos(th)
    y=yc+r*sin(th)
    plot(round(x),round(y),color)
    th=th+dth
End
    
```

x	Round(x)	y	Round(y)	th	Plot (x,y)
15	15	10	10	0	(15,10)
14.9	15	10.9	11	0.2	(15,11)
14.6	15	11.9	12	0.4	(15,12)
14.1	14	12.8	13	0.6	(14,13)
13.4	13	13.5	14	0.8	(13,14)
12.7	13	14.2	14	1	(13,14)
11.8	12	14.6	15	1.2	(12,15)
:	:	:	:	:	:
:	:	:	:	:	:
14.9	15	9.5	10	6.4	(15,10)

