Computer Graphics

Seven Chapter
part 1

2025-2026

Window to Viewport Mapping

Window:

1. A world-coordinate area selected for display is called a window.
2. In computer graphics, a window is a graphical control element.
3. It consists of a visual area containing some of the graphical user interface of the program it belongs to and is framed by a window decoration.
4. A window defines a rectangular area in world coordinates. You can define the window to be larger than, the same size as, or smaller than the actual range of data values, depending on whether you want to show all of the data or only part of the data.

5. Window defines what is to be viewed

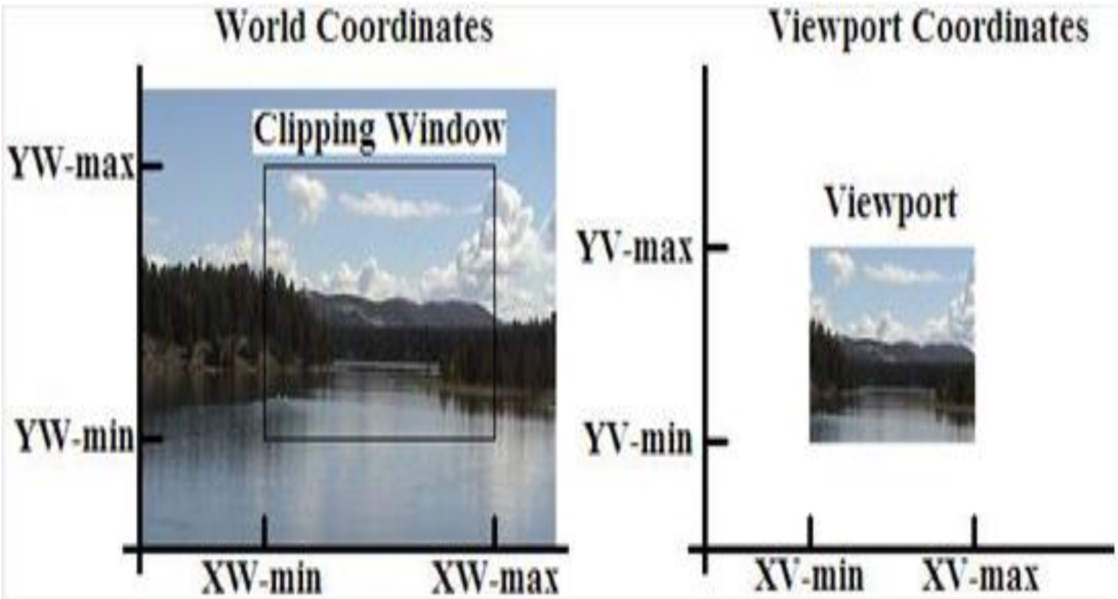
Viewport:

1. An area on a display device to which a window is mapped is called a viewport.
2. A viewport is a polygon viewing region in computer graphics. The viewport is an area expressed in rendering-device-specific coordinates, e.g. pixels for screen coordinates, in which the objects of interest are going to be rendered.
3. A viewport defines in **normalized coordinates** a rectangular area on the display device where the image of the data appears. You can have your graph take up the entire display device or show it in only a portion, say the upper-right part.

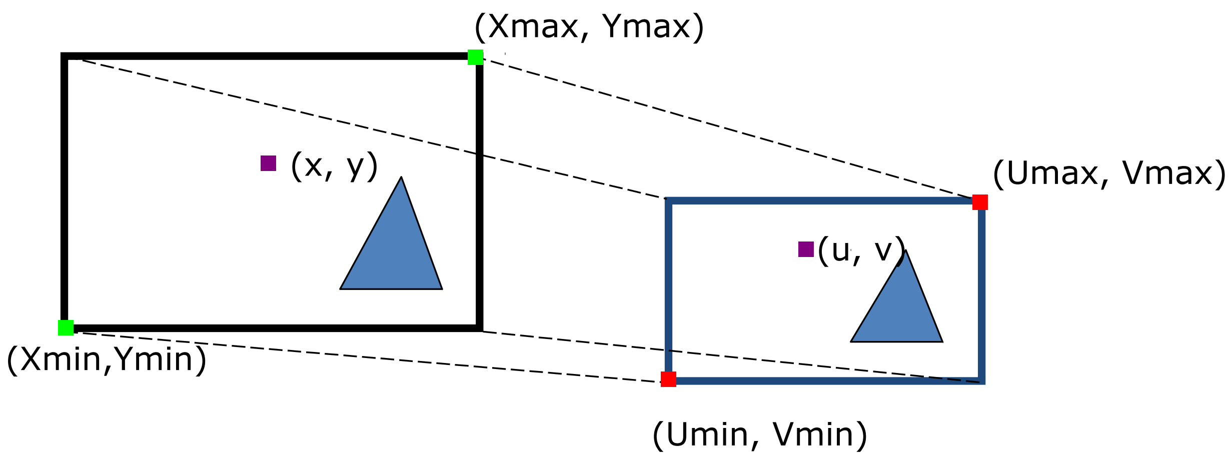
4. Viewport defines where the window to be displayed

Window to viewport transformation

1. Window-to-Viewport transformation is the process of transforming a two-dimensional, world-coordinate scene to device coordinates.
2. In particular, objects inside the world or clipping window are mapped to the viewport. The viewport is displayed in the interface window on the screen.
3. In other words, the clipping window is used to select the part of the scene that is to be displayed. The viewport then positions the scene on the output device.
4. **Example:**



This transformation involves developing formulas that start with a point in the world window, say (x, y).



(x, y) ⟶ **(u, v)**

$$\frac{x - x_{min}}{x_{max} - x_{min}} = \frac{u - u_{min}}{u_{max} - u_{min}} \quad \text{----------(1)}$$

$$\frac{y - y_{min}}{y_{max} - y_{min}} = \frac{v - v_{min}}{v_{max} - v_{min}} \quad \text{----------(2)}$$

By rewriting this relationship, we get the following formula:

$$u = c_1 x + c_2 \quad \text{----------(3)}$$

$$c_1 = \frac{u_{max} - u_{min}}{x_{max} - x_{min}} \quad \text{---------(4)}$$

$$c_2 = u_{min} - c_1 x_{min} \quad \text{--------(5)}$$

$$v = d_1 y + d_2 \quad \text{----------(6)}$$

$$d_1 = \frac{v_{max} - v_{min}}{y_{max} - y_{min}} \quad \text{----------(7)}$$

$$d_2 = v_{min} - d_1 y_{min} \quad \text{----------(8)}$$

Example1:

A normalized window has left and right boundaries of (-0.05 to +0.05) and lower and upper boundaries of (0.1 to 0.2). the viewport window left and right is (250,550) and lower to upper is (100,400),find the coordinate of any point (u,v) in the viewport window.

Solution

Window(x_{min}=-0.05, x_{max}=+0.05 , y_{min}=0.1, y_{max}=0.2)

Viewport (u_{min}=250 , u_{max}=550, , v_{min}=100, v_{max}=400)

u=$c1x +$ c2

$$c1=\frac{u_{max} - u_{min}}{x_{max} - x_{min}}$$

$$c1=\frac{(550-250)}{0.05-(-0.05)}=300/0.1=3000$$

c2=u_{min}-c1x_{min}

=250-3000(-0.05) =250+150 =400

u=3000x+400

v=d1y+d2

$$d1 = \frac{v_{max} - v_{min}}{y_{max} - y_{min}}$$

$$d1 = \frac{(400-100)}{(0.2-0.1} =300/0.1=3000$$

d2=vmin-d1ymin

=100-3000(0.1)

=-200

v=3000y-200

Example2:

A normalized window has left(x_{min}=10) and right(x_{max}=50) boundaries and lower(y_{min}=5) and upper(y_{max}=30) boundaries .the viewport window left(u_{min}=25) and right(u_{max}=75) and lower(v_{min}=25) to upper (v_{max}=75) find the coordinate of any point (u,v) in the viewport window

$u = c1x + c2$

$$c1 = \frac{u_{max} - u_{min}}{x_{max} - x_{min}}$$

c1 =(75-25)/(50-10)

c1=50/40=1.25

$c2 = u_{min} - c1x_{min}$

=25-1.25*10

= 25-12.5=12.5

u=1.25x+12.5

v=d1y+d2

$$d1 = \frac{v_{max} - v_{min}}{y_{max} - y_{min}}$$

= (75-25)/ (30-5)

d1=50/25=2

$d2 = v_{min} - d1y_{min}$

=25-2*5

=25-10=15

v=2y+15

Window to Viewport Transformation N

We can express these two formula for computing (u,v) from (x,y) by term:

(translate-scale-translate)

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} . N$$

$N=T_2ST_1$

1. T_1 is the translation matrix about window origin :

$$T1 = \begin{bmatrix} 1 & 0 & -xmin \\ 0 & 1 & -ymin \\ 0 & 0 & 1 \end{bmatrix}$$

2. S is the scaling transformation matrix:

$$S = \begin{bmatrix} \frac{umax-umin}{xmax-xmin} & 0 & 0 \\ 0 & \frac{vmax-vmin}{ymax-ymin} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. T2 is the translation matrix position of the viewport :

$$T2 = \begin{bmatrix} 1 & 0 & umin \\ 0 & 1 & vmin \\ 0 & 0 & 1 \end{bmatrix}$$

Example1:

A normalized window has left and right boundaries of (-0.05 to +0.05) and lower and upper boundaries of (0.1 to 0.2). the viewport window left and right is (250,550) and lower to upper is (100,400),find the transformation N.

Solution **N=T$_2$ST$_1$**

$$T1 = \begin{bmatrix} 1 & 0 & -xmin \\ 0 & 1 & -ymin \\ 0 & 0 & 1 \end{bmatrix}$$

$$T1 = \begin{bmatrix} 1 & 0 & -(-0.05) \\ 0 & 1 & -0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{umax - umin}{xmax - xmin} & 0 & 0 \\ 0 & \frac{max - \quad min}{ymax - ymin} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T2 = \begin{bmatrix} 1 & 0 & umin \\ 0 & 1 & vmin \\ 0 & 0 & 1 \end{bmatrix}$$

$$T2 = \begin{bmatrix} 1 & 0 & 250 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 & 250 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(-0.05) \\ 0 & 1 & -0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 2

Specify individually the translation and scaling matrices required to transform a 2D window of [Xmin=-234, Ymin=156] and [Xmax=66, Ymax=456] to a display viewport of [Umin=45, Vmin=35] and [Umax=245, Vmax=185].

Solution N=T_2ST_1

$$1 = \begin{bmatrix} 1 & 0 & -xmin \\ 0 & 1 & -ymin \\ 0 & 0 & 1 \end{bmatrix}$$

$$1 = \begin{bmatrix} 1 & 0 & -(-234) \\ 0 & 1 & -156 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{umax-umin}{xmax-xmin} & 0 & 0 \\ 0 & \frac{vmax-vmin}{ymax-ymin} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{245-45}{66+234} & 0 & 0 \\ 0 & \frac{185-35}{456-156} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2 = \begin{bmatrix} 1 & 0 & umin \\ 0 & 1 & vmin \\ 0 & 0 & 1 \end{bmatrix}$$

$$2 = \begin{bmatrix} 1 & 0 & 45 \\ 0 & 1 & 35 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 & 250 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(-0.05) \\ 0 & 1 & -0.1 \\ 0 & 0 & 1 \end{bmatrix}$$