Computer Graphics

Sixth Chapter
Part 2

2025-2026

Matrix representation of transformation

Many graphic applications involves sequence of geometric transformations. For example , animation transformation which is require an object to be translated and rotated at each increment of the motion.

- Transformation can be represented as a product of the row vector [x,y] and a 2x2 matrix accept for the translation.
- Transformations can be combined using matrix multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Matrices are convenient to represent a sequence of transformations

1- Translation matrix T(tx , ty)

We can represent the translation transformation as follows:

P' = P+T, $P = \begin{bmatrix} x \\ y \end{bmatrix}, \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix} \dashrightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

2- Scaling matrix

- If a point P $\begin{bmatrix} x \\ y \end{bmatrix}$ is being a 2x1 vector. If we multiply it by 2x2 matrix S=$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$. We will obtain another 2x1 vector which we can interpret as another point:
  P'= S . P
  $\begin{bmatrix} x' \\ y' \end{bmatrix}$= $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$. $\begin{bmatrix} x \\ y \end{bmatrix}$

- What will happen if we transfer every point by means of multiplication by S and display the result:
  1- If S is the Identity matrix: S=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$→ No change
  2- If S=$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ then $\begin{bmatrix} x' \\ y' \end{bmatrix}$= $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. $\begin{bmatrix} x \\ y \end{bmatrix}$= $\begin{bmatrix} 2x \\ y \end{bmatrix}$
  That mean:
  - every new x coordinate would be twice as large as the old value of vertical lines.
  - x coordinate would be twice as width and the same tall.

  3- If S= $\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$→ shrink all x coordinate (shrink the width with the same tall)

  <u>Examples:</u>

  1- Stretch the image/object to twice and then compress it to one-half of the new width?

  P'= (S_1S_2). P

  S_1S_2=$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $\begin{bmatrix} x \\ y \end{bmatrix}$ → identity matrix then no change

2- Make an image twice as width and twice as tall?

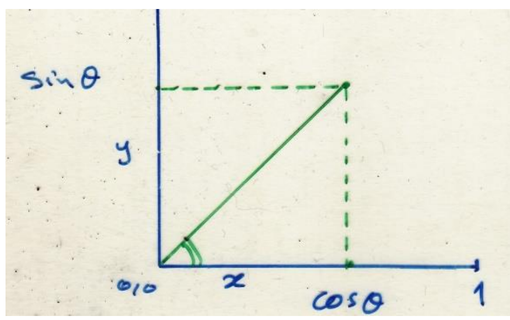
P'= (S_1S_2). P

$(S_1S_2)\ .\ P=\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}.\ P \quad ---\rightarrow \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}.\begin{bmatrix}x\\y\end{bmatrix}$

2- Rotation matrix

If a line segment have two endpoints (0,0) and (x,y), and length $L = \sqrt{x^2 + y^2}$:

- The ratio of the height of the (x,y) endpoint with x-axis have the y coordinates value and the length of the segment will be the *Sin* of the angle: $\text{Sin}(\theta) = \frac{y}{\sqrt{x^2+y^2}}$

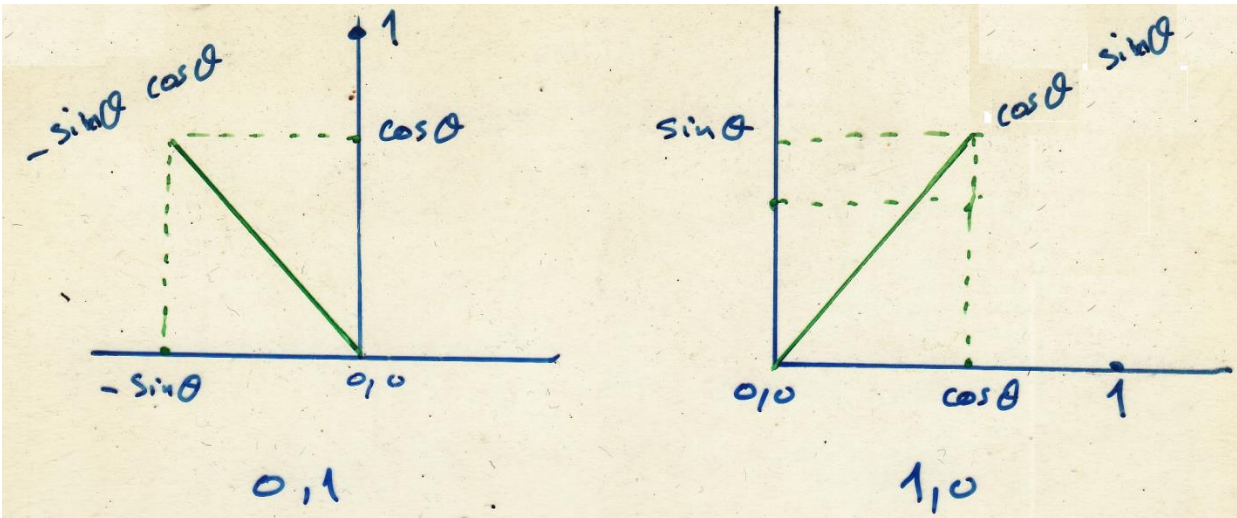


- The ratio of the distance of the (x,y) endpoint with y-axis have the x coordinates value and the length of the segment will be the *Cosine* of the angle: $\text{Cos}(\theta) = \frac{x}{\sqrt{x^2+y^2}}$
- If L=1 then Sin(θ)=y and Cos(θ)=x
- If we rotate the point (1,0) in counterclockwise by an angle θ, it becomes (Cos(θ), Sin(θ)) so:

$$\begin{bmatrix}\cos(\theta)\\ \sin(\theta)\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}a\\c\end{bmatrix}$$

- If we rotate the point (0,1) in counterclockwise by an angle θ, it becomes (-Sin(θ), Cos(θ)) so:

$$\begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$$



From these equations we can see the values of a,b,c,d needed to form the rotation matrix:

$$R = \begin{bmatrix} Cos(\theta) & -Sin(\theta) \\ Sin(\theta) & Cos(\theta) \end{bmatrix}$$

Example:

Rotate the point (2,3) in counterclockwise by an angle $\pi/6$?

The rotation matrix is:

$$R = \begin{bmatrix} Cos\left(\frac{\pi}{6}\right) & -Sin(\pi/6) \\ Sin(\pi/6) & Cos(\pi/6) \end{bmatrix} = \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix}$$

Then the rotated point would be :

P' = R . P

$$P' = \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3.232 \\ 3.598 \end{bmatrix}$$

Notes:

- We can rotate an entire line segment by rotating both endpoints which specify it.

- The rotation matrix for an angle θ in clockwise would be:

$$\begin{bmatrix} Cos(-\theta) & -Sin(-\theta) \\ Sin(-\theta) & Cos(-\theta) \end{bmatrix} = \begin{bmatrix} Cos(\theta) & Sin(\theta) \\ -Sin(\theta) & Cos(\theta) \end{bmatrix}$$

H.W-1\: Rotate the triangle (5,2),(1,1),(0,0) by rotation in counterclockwise with angle 45° about origin?

H.W-2

a- Find the matrix that represent rotation of an object by 30° about the origin?

b- what are the new coordinates of the point p(2,-4) after the rotation?

<u>Homogenous coordinate in transformation matrix</u>

- Why Homogeneous Coordinates?
    1. Mathematicians commonly use homogeneous coordinates as they allow scaling factors to be removed from equations.
    2. Using homogeneous coordinates allows us use matrix multiplication to calculate transformations – extremely efficient!
    3. Homogeneous coordinates seem unintuitive, but they make graphics operations much easier.
    4. Since a 2x2 matrix representation of translation does not exist!!. So by using a homogenous coordinate system then we can represent *2x2 translation transformation* as a matrix multiplication.
    5. It provides a consistent, uniform way of handling *affine transformations*. 2D affine transformations always have a bottom row of **[0 0 1].**

       An "affine point" is a "linear point" with an added w-coordinate which is always 1:

$$p_{aff} = \begin{bmatrix} p_{lin} \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Applying an affine transformation gives another affine point.

- A point (x, y) can be re-written in homogeneous coordinates as (xw, yw,w)
  - The homogeneous parameter w is a non-zero value such that x and y coordinates can easily be recovered by dividing the first and second numbers by the third.

$$x = \frac{xw}{w} \qquad\qquad y = \frac{yw}{w}$$

  - We can then write any point (x, y) as :

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} w.x \\ w.y \\ w \end{bmatrix}, w \neq 0$$

  - We can conveniently choose w = 1 so that (x, y) becomes:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- In homogeneous coordinates the **scaling matrix** as follows:

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \rightarrow \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} xw \\ yw \\ w \end{bmatrix} = \begin{bmatrix} S_x xw \\ S_y yw \\ w \end{bmatrix}$ → divide by w then $\begin{bmatrix} S_x x \\ S_y y \end{bmatrix}$ is the correctly scaled point.

- The counterclockwise **rotation matrix** is

$$\begin{bmatrix} Cos(\theta) & -Sin(\theta) \\ Sin(\theta) & Cos(\theta) \end{bmatrix}$$

Using homogeneous coordinates we get: $\begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Applying it to the point (x,y) with homogeneous (xw,yw,w) gives:

$$R.P = \begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} xw \\ yw \\ w \end{bmatrix}$$

- The homogeneous coordinate **translation matrix** of t_x , t_y is

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$T.P = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} xw \\ yw \\ w \end{bmatrix} = \begin{bmatrix} (xw + t_x w) \\ (yw + t_y w) \\ w \end{bmatrix} \rightarrow$$

divide by w then we get the point P= $\begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$ is the correctly translated point.

In general:

1- Translation

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2- Rotation

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = \begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3- Scaling

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **<u>Rotation about an arbitrary point</u>**
  - The homogeneous coordinate transformation matrix for counterclockwise rotation about point (x_c , y_c) is done by three steps as follows:

1- Translation to the origin

$$T_1 = \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2- Rotation about the origin

$$R = \begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \bar{\bar{x}} \\ \bar{\bar{y}} \\ 1 \end{bmatrix} = \begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix}$$

3- Translation back to its correct position

$$T_2 = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \bar{\bar{\bar{x}}} \\ \bar{\bar{\bar{y}}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\bar{x}} \\ \bar{\bar{y}} \\ 1 \end{bmatrix}$$

Composite Transformations

- Matrices are a convenient and efficient way to represent a sequence of transformations.
- Matrix multiplication is not commutative so that the order of transformations is important.
- What if we want to rotate and translate?

To rotate a point $(T_2\ (\mathrm{R}\ (T_1 \begin{bmatrix} xw \\ yw \\ w \end{bmatrix})))$

- Now we must form an overall transformation matrix as follows:-

$$(T_2(\mathrm{R}\ T_1)) \begin{bmatrix} xw \\ yw \\ w \end{bmatrix}$$

$$T_2\mathrm{R}\ T_1 = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cos(\theta) & -Sin(\theta) & 0 \\ Sin(\theta) & Cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cos(\theta) & -Sin(\theta) & -x_c \mathrm{Cos}(\theta) + y_c Sin(\theta) \\ Sin(\theta) & Cos(\theta) & -x_c \sin(\theta) - y_c Cos(\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Cos(\theta) & -Sin(\theta) & -x_c Cos(\theta) + y_c Sin(\theta) + x_c \\ Sin(\theta) & Cos(\theta) & -x_c Sin(\theta) - y_c Cos(\theta) + y_c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Cos(\theta) & -Sin(\theta) & x_c(1 - Cos(\theta)) + y_c Sin(\theta) \\ Sin(\theta) & Cos(\theta) & y_c(1 - Cos(\theta)) - x_c Sin(\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Rotate line segment by 45 degrees about endpoint **a** and note that tx=3?

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(45) & -\sin(45) & 0 \\ \sin(45) & \cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ 1 \end{bmatrix} = \begin{bmatrix} a'_x \\ a'_y \\ 1 \end{bmatrix}$$

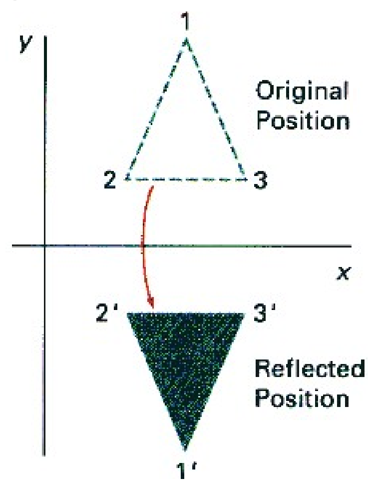
H.W Rotate the triangle (5,2),(1,1),(0,0) by rotation in counterclockwise with angle 45° about fixed point (-1,-1)?

Reflection

- Transformation that produces a mirror image of an object is called *reflection*.
- Image is generated relative to an axis of reflection by rotating the object **180°** about the reflection axis.
- A Common reflections as follows:

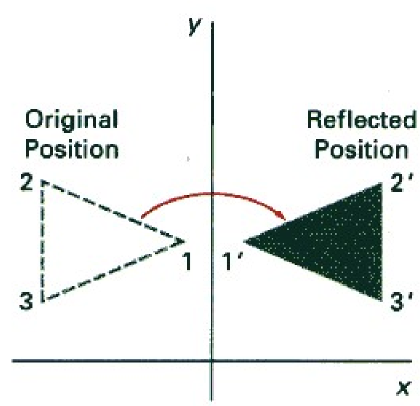
1- Reflection about the line y=0 (the x-axis) is accomplished with the transformation matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$ → Reflection in the x-axis



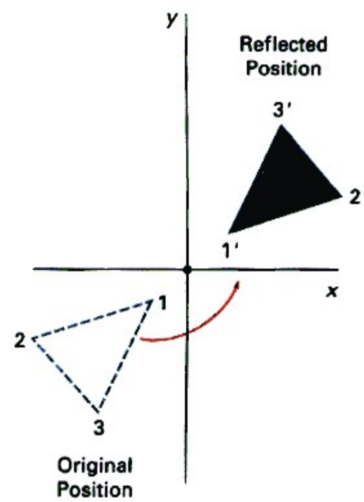
2- Reflection about the line x=0 (the y-axis) is accomplished with the transformation matrix:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$ → Reflection in the y-axis



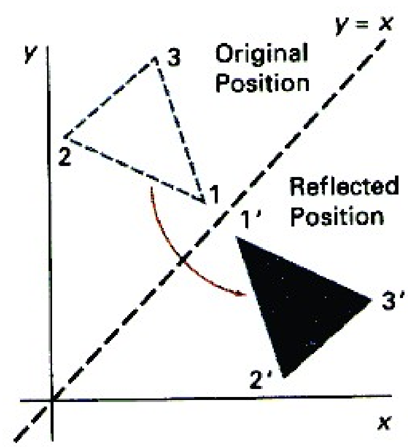
3- Reflection about the origin which is equivalent to the rotation matrix R(θ) with θ=180°

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$ → Reflection about the origin



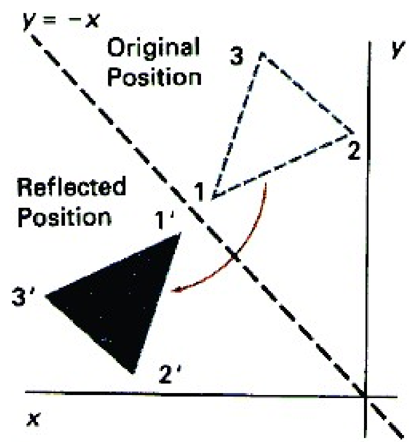
4- Reflection in the line y=x (diagonal line)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$ → Reflection about the diagonal line y=x



5- Reflection about the diagonal line y=-x

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$ → Reflection about the diagonal line y=-x



Coordinate plane rules:

Over the x-axis : (x, y) → (x, −y)

Over the y-axis : (x, y) → (−x, y)

Through the origin : (x, y) → (−x, −y)

Over the line y = x : (x, y) → (y, x)

Over the line y = -x : (x, y) → (-y, -x)

Example1: Given a triangle with coordinate points A(3, 4), B(6, 4) and C(5, 6). Apply the reflection transformation:

1- about the x-axis

2- about the y-axis

3- about the origin

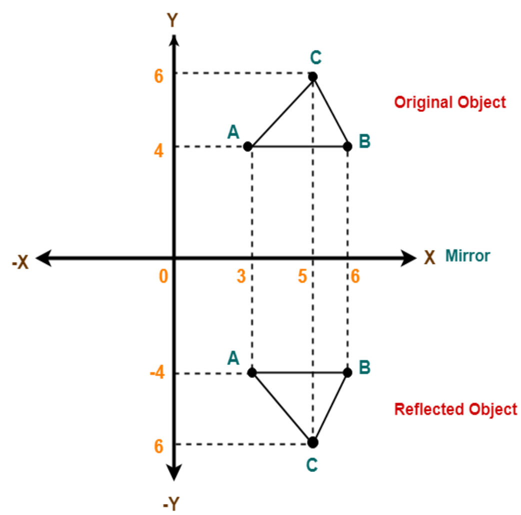
4- about the diagonal line y=x

5- about the diagonal line y=-x

Sol: \ 1- about the x-axis

y=0 ; $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow$ reflection matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 6 & -4 \\ 5 & -6 \end{bmatrix}$$



OR

Sol: \ 1- about the x-axis

y=0 ; $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow$ reflection matrix

$$[3 \;\; 4] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [3 \; -4]$$

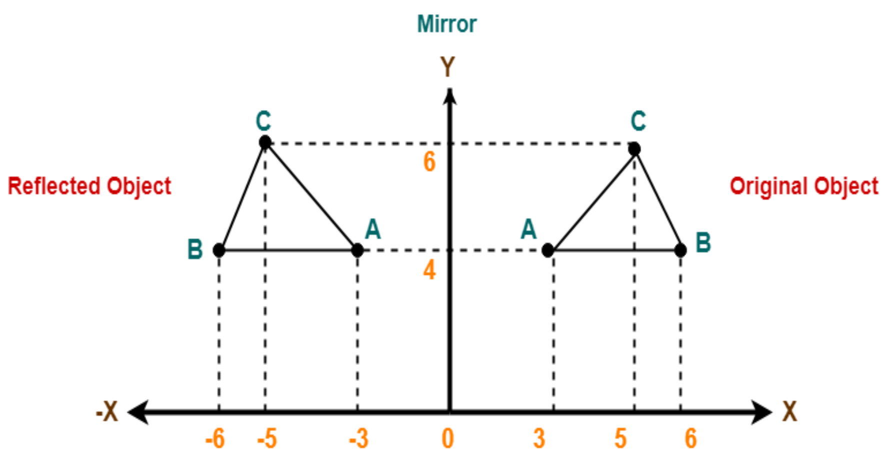
$$[6 \;\; 4] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [6 \; -4]$$

$$[5 \;\; 6] \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [5 \; -6]$$

Sol: \ 2- about the y-axis

x=0 ; $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow$ reflection matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -6 & 4 \\ -5 & 6 \end{bmatrix}$$



Sol: \ 3- about the origin

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow$ reflection matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ -6 & -4 \\ -5 & -6 \end{bmatrix}$$

Sol: \ 4- about the diagonal line y=x

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 4 & 6 \\ 6 & 5 \end{bmatrix}$$

Sol: \ 5- about the diagonal line y=-x

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \rightarrow \text{ reflection matrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ -4 & -6 \\ -6 & -5 \end{bmatrix}$$

Shear

- Distorting or changing the shape of an object by differentially moving some of its vertices as if the object internal layers are sided over each other is called **Shear.**
- Shears either shift coordinates x values or y values
- Similar to scaling, the shear transformation requires two parameters (s_x, s_y) not on the main diagonal of the transformation matrix but on the other two positions.

$$\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix}$$

- Applying a shear transformation sh(s_x, s_y) to point (x,y) yields the point ($\bar{x}$, $\bar{y}$) with the new coordinates.

$$[\bar{x} \quad \bar{y}] = [x \quad y]\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix} = [x + y * s_y \qquad x * s_x + y]$$

- Shear transformations are carried out with respect to the origin of the coordinate system so that an object that is not centered around the origin will not only be deformed by a shear transformation but also shifted.
- If s_x=0 the shear take place in the x direction and in the y direction if s_y=0

Example: Consider the square whose vertices are: A(0,0), B(1,0), C(1,1), and D(0,1). Perform the shear transformation:

1- In the x direction
2- In the y direction
3- In both direction

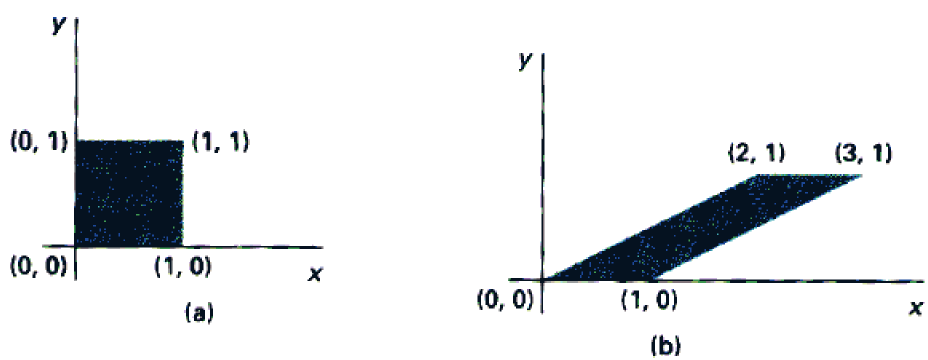
Where s_x=1 and s_y=2

Sol:

1- Shear in the x direction:

s_x=0 ; $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ → shear matrix

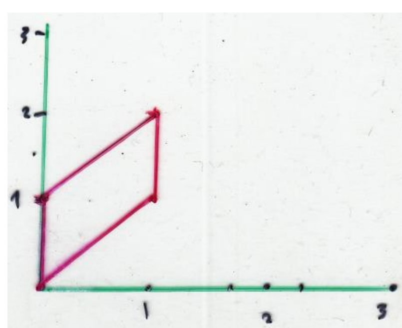
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$



2- Shear in the y direction:

s_y=0

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ → shear matrix



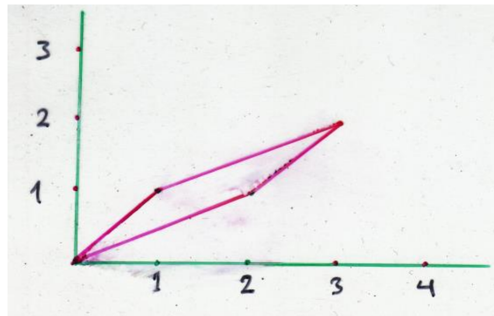
$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$$

3- Shear in both direction:

s_x=1 s_y=2

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ → shear matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \\ 1 & 1 \\ 3 & 2 \end{bmatrix}$$



H.W Perform a shear transformation for the shape A(3,0), B(3,3), C(5,3), and D(0,5)

1- In the x-direction
2- In the y-direction
3- In the both direction
Where s_x=2 s_y=1