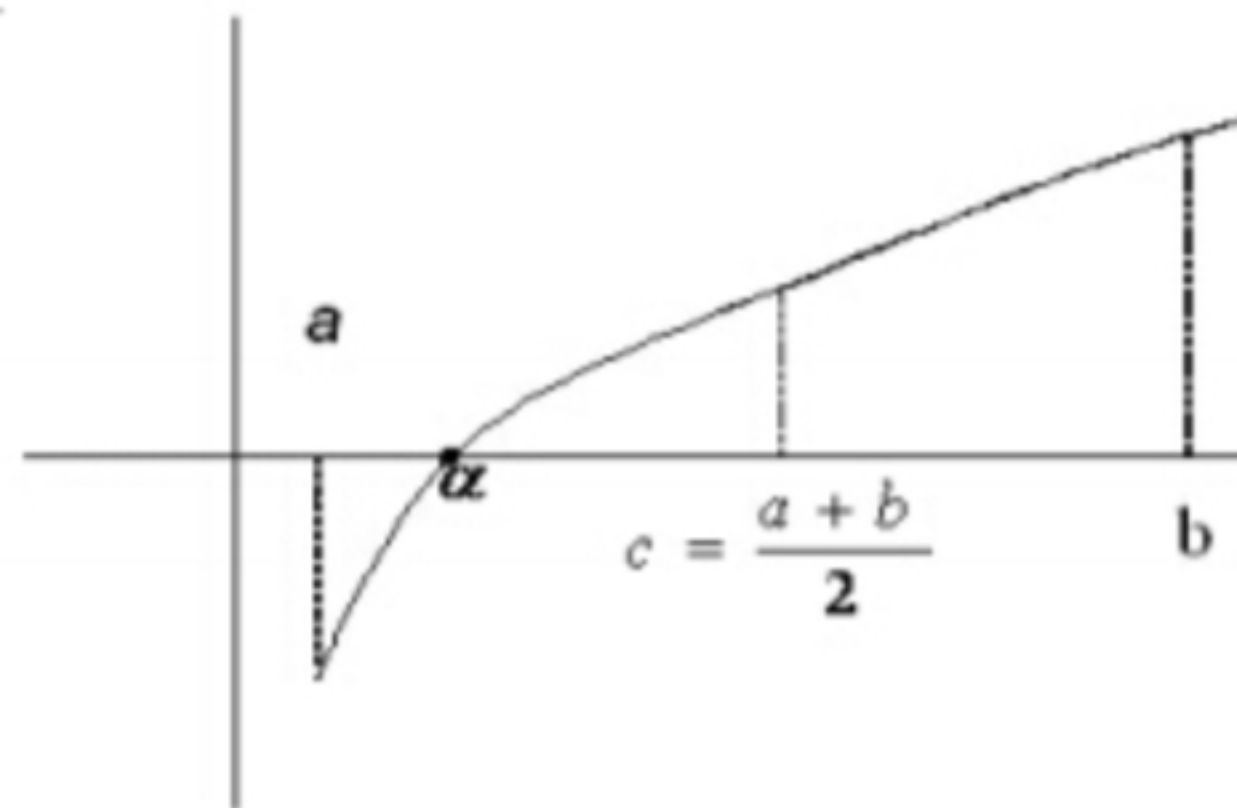


# Numerical Analysis

## Solution of Nonlinear equations

### a. Bisection Method

The Bisection (or half interval ) method is an algorithm for location the roots of a function. If the initial interval is between  $x = a$  and  $x = b$  we know that  $f(a)$  and  $f(b)$  are of different signs.  $f(a).f(b) < 0$  (-ve value) and  $f(x) = 0$  has at . last one root on the interval.



A simple algorithm for the bisection method consists of the following steps:

- 1- Choose lower  $x = a$  and upper  $x = b$ . We must checked by ensuring that  $[f(a).f(b) < 0]$ .
- 2- Define  $c = (a + b) / 2$
- 3- Make the following evolutions:
  - a. If  $f(a).f(c) < 0$  . Therefore, set  $b = c$  and return to step 2.

- b. If  $f(a).f(c) > 0$  . Therefore, set  $a = c$  and return to step 2.
- c. If  $f(a).f(c) = 0$  , the root equals  $c$ , terminate the computation.

**Condition of c stop**

Using some mathematical formulas to stop doing calculations for this method.

$$1- |x_{i+1} - x_i| < \epsilon$$

$$2- |f(x_i)| < \epsilon$$

$$3- \frac{|x_{i+1} - x_i|}{|x_i|} < \epsilon$$

**Example:** by the Bisection method, find the largest root for  
 $f(x) = x - \cos(x)$ ,  $\epsilon = 0.020$

**Solution: RAD**

x	0	1
f(x)	-	+

x	0	1	0.5	0.75	0.625	0.688	0.719	0.735
f(x)	-1	0.460	-0.378	0.018	-0.186	-0.085	-0.033	-0.006

$$\because |x_7 - x_6| = |0.735 - 0.719| = 0.016 < \epsilon = 0.02$$

$$x_6 = 0.735 \text{ is a root}$$

**Example:** By using Bisection method find the first positive root

$$f(x) = x^6 - x - 1 \text{ with } \epsilon = 0.001$$

Solution:

x	0	1	2
f(x)	-	-	+

n	a	b	c	b - c	f(c)
1	1	2	1.5	0.5	8.8906
2	1	1.5	1.25	0.25	1.5647
3	1	1.25	1.125	0.125	-0.0977
4	1.125	1.25	1.1875	0.0625	0.6167
5	1.125	1.1875	1.15625		0.2333
6	1.125	1.15625	1.140625		0.0616
7	1.125	1.140625	1.1328125		-0.0196
8	1.1328125	1.140625	1.13671875		0.0206
9	1.1328125	1.13671875	1.134765625		0.0004
10	1.1328125	1.134765625	1.133789063	0.001	-0.0096

There are several advantages to the Bisection method:

- 1- The method guaranteed to converge.
- 2- The error bound is guaranteed to decrease by half with each iteration.

The principle disadvantage of the Bisection method is that:

It generally converges slower than other methods.

H.W :

1-  $x^3 - x^2 - x - 1 = 0$  with  $[1, 2]$ ,  $\epsilon = 0.001$

2-  $x^3 + 4x^2 - 10 = 0$  with  $[1, 2]$ ,  $\epsilon = 0.001$

### **b. False Position Method**

Similarly to the bisection method, the false position or called linear interpolation method starts with the initial solution interval  $[a,$

b] that is believed to contain the solution of  $f(x) = 0$ . Approximating the curve of  $f(x)$  on  $[a, b]$  by a straight line connecting the two points  $(a, f(a))$  and  $(b, f(b))$ , it guesses that the solution may be the point at which the straight line crosses the x-axis:

$$\frac{f(a)}{x-a} = \frac{f(b)}{x-b}$$

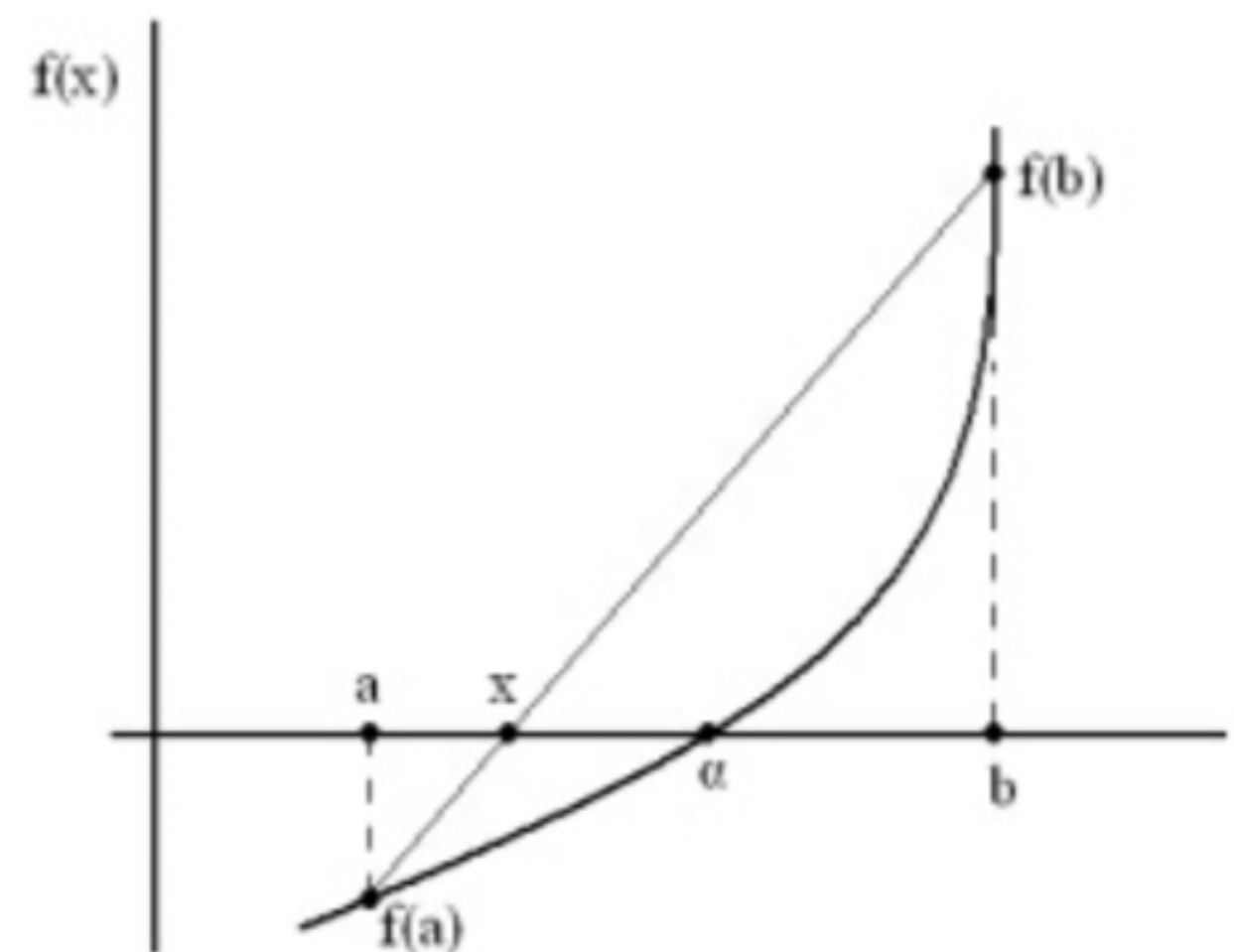
$$x = b - \frac{f(b)}{f(b)-f(a)}(b-a)$$

### Algorithm

**Input:**  $a, b, f(a)$ , and  $f(b)$

**Compute:**

- $x = b - \frac{f(b)}{f(b)-f(a)}(b-a);$
- $f(x);$
- If  $f(a) * f(x) < 0$   
 $b = x;$   
 $f(b) = f(x);$
- If  $f(a) * f(x) > 0$   
 $a = x;$



$$f(a) = f(x);$$

- If  $f(a) \cdot f(x) = 0$ , the root  $(a) = x$

end;

**Output:**  $x, f(x)$

**Example:** By the false position method find approximate value of the root for  $f(x) = e^x - 2$  with  $\epsilon = 0.005$

**Solution:**

x	0	1
f(x)	-	+

$$a = 0 \Rightarrow f(a) = -1$$

$$b = 1 \Rightarrow f(b) = 0.718$$

$$x = b - \frac{f(b)}{f(b) - f(a)}(b - a)$$

$$x_2 = 1 - \frac{0.718}{0.718 + 1}(1 - 0) = 0.582$$

$$f(x_2) = e^{0.582} - 2 = -0.21$$

$$f(a) * f(x_2) > 0 \Rightarrow x_2 = a$$

$$x_3 = 1 - \frac{0.718}{0.718 + 0.21}(1 - 0.5821) = 0.677$$

$$f(x_3) = e^{0.677} - 2 = -0.032$$

$$f(a) * f(x_3) > 0 \Rightarrow x_3 = a$$

$$x_4 = 1 - \frac{0.718}{0.718 + 0.032} (1 - 0.677) = 0.691$$

$$f(x_4) = e^{0.691} - 2 = -0.004$$

since  $|f(x)| = 0.004 < \epsilon = .005$

$\therefore x = 0.691$  is a root.

### **Example:**

Solve the following equation using false position method

$$f(x) = x^2 - x - 1 \quad \text{with interval } [1,2]$$

### **Solution:**

n	a	b	x	f(x)	$\epsilon = x_{i+1} - x_i$
1	1	2	1.5	-0.25	1.6-1.5=0.1
2	1.5	2	1.6	-0.04	0.0154
3	1.6	2	1.6154	$-5.8828 \cdot 10^{-3}$	0.0022
4	1.6154	2	1.6176	$-9.7024 \cdot 10^{-4}$	0.0004
5	1.6176	2	1.6180	$-7.500 \cdot 10^{-5}$	0
6	1.6180	2	1.6180	$-7.500 \cdot 10^{-5}$	0

$\therefore x = 1.6180$  is a root.

There are several advantages of the false position method:

- 1- The false position method is faster than the bisection method.
- 2- It includes a test to ensure that the root is bracketed between successive iterations.

The disadvantage of the false position method:

Although the false position method aims to improve the convergence speed over the bisection method, it cannot always achieve the goal, especially when the curve of  $f(x)$  on  $[a, b]$  is not well approximated by a straight line as depicted in the following figure:

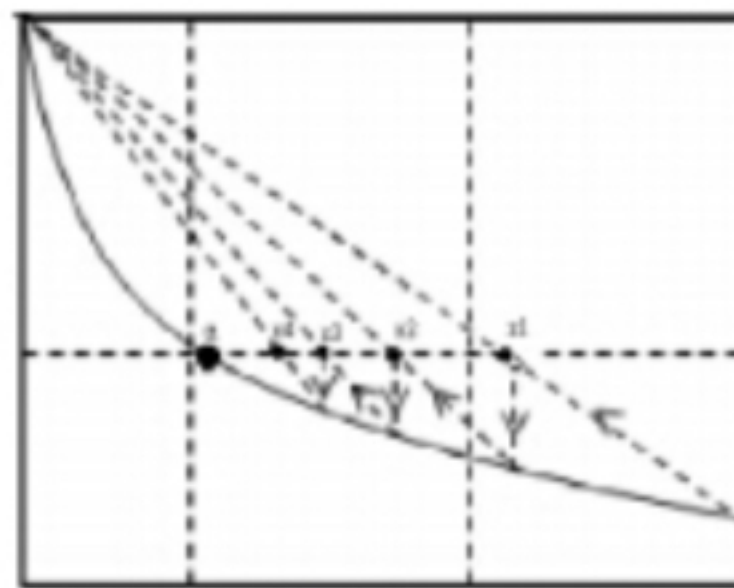


Figure: Solve the equation  $(\tan(\pi - x) - x = 0)$  by false position method.

H.W:

1-  $x^3 - x^2 - x - 1 = 0$  with  $[1, 2]$ ,  $\epsilon = 0.0001$

2-  $\exp(x) - 2x^2 = 0$ , At a.  $0 \leq x \leq 1$  b.  $3 \leq x \leq 4$ ,  $\epsilon = 0.00005$