d.Secant Method

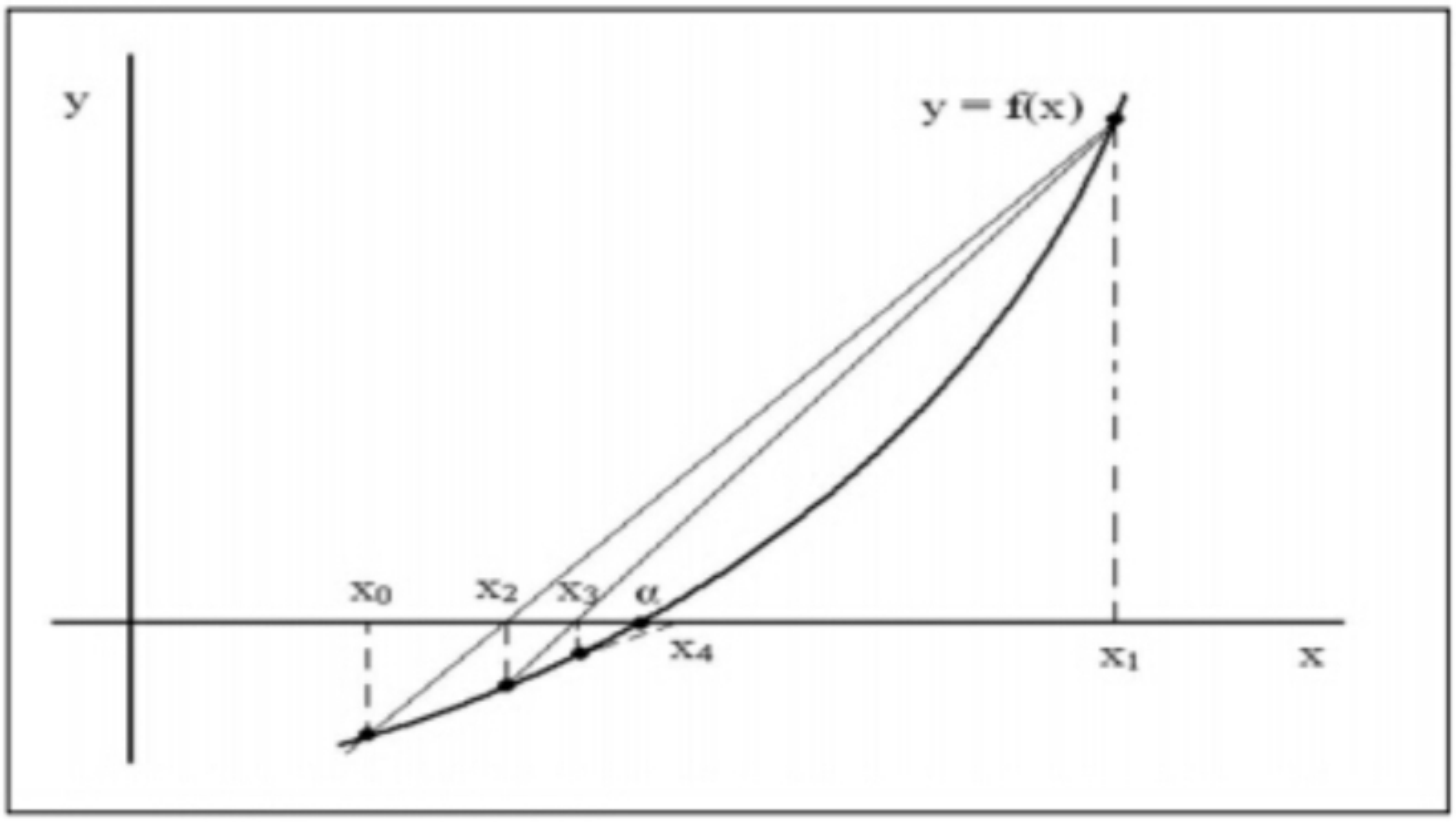
The secant method can be regarded as a modification of the Newton method in the sense that the derivative is replaced by a difference approximation based on the successive estimates

$$f'(x) \cong \frac{f(x_n) - f(x_{n-1})}{(x_n - x_{n-1})}$$

This approximation can be substituted into the equation of Newton method to yield the following iterative equation:-

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

This technique is called the Secant method. It start with the two initial approximations x_0 and x_1, the approximation x_2 is the x-intercept of the line joining $(x_0, f(x_0))$ and $(x_1, f(x_1))$. The approximation x_3 is the x-intercept of the line joining $(x_1, f(x_1))$ and $(x_2, f(x_2))$ and so on, shows the figure below:



Conditions of stop

Using some mathematical formulas to stop doing calculations for this method.

1- $|x_{i+1} - x_i| < \epsilon$

2- $|f(x_i)| < \epsilon$

<u>Example</u>:

Solve the equation below by using the secant method

$f(x) = x^2 - x - 1 = 0$ with $\epsilon = 0$

<u>Solution:</u>

x	0	1	2
$f(x)$	-	-	+

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_0 = 1 \Rightarrow f(x_0) = f(1) = 1^2 - 1 - 1 = -1$$

$$x_1 = 2 \Rightarrow f(x_1) = f(2) = 2^2 - 2 - 1 = 1$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 2 - \frac{1 * (2 - 1)}{(1 + 1)} = 1.5$$

$$f(1.5) = (1.5)^2 - 1.5 - 1 = -0.25$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 1.5 - \frac{-0.25 * (1.5 - 2)}{(-0.25 - 1)} = 1.5 - \frac{0.125}{-1.25}$$

$$= 1.5 + 0.1 = 1.6$$

$$f(1.6) = (1.6)^2 - 1.6 - 1 = -0.04$$

$$x_4 = x_3 - \frac{f(x_3)(x_3-x_2)}{f(x_3)-f(x_2)} = 1.6 - \frac{-0.04*(1.6-1.5)}{(-0.04+0.25)} = 1.6 - \frac{-0.004}{0.21} = 1.6 + 0.019 = 1.619$$

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n	x_n	$f(x_n)$	$x_{n+1} - x_n$
0	1	-1	1
1	2	1	-0.5
2	1.5	-0.25	0.1
3	1.6	-0.04	0.019
4	1.619	$2.161*10^{-3}$	-0.001
5	1.618	$-7.6*10^{-5}$	0
6	1.618	$-7.6*10^{-5}$	

Then $x_6 = 1.618$ is a root.

The advantage of the secant method:

1. Linear convergence near multiple roots

2. No derivative needed

3. Works for complex functions

The disadvantage of the secant method:

1. Iterates may diverge
2. No easy error bound

H.W:

Solve the same H.w of false position method