EXAMPLE 6 Solve the inequality and show the solution set on the real line:

(a) $|2x - 3| \leq 1$ **(b)** $|2x - 3| \geq 1$

Solution

(a)

$$\begin{aligned} |2x - 3| &\leq 1 \\ -1 \leq 2x - 3 &\leq 1 && \text{Property 8} \\ 2 \leq 2x &\leq 4 && \text{Add 3.} \\ 1 \leq x &\leq 2 && \text{Divide by 2.} \end{aligned}$$

The solution set is the closed interval [1, 2] (Figure 1.4a).

(b)

$$|2x - 3| \geq 1$$

$$\begin{aligned} 2x - 3 \geq 1 \quad &\text{or} \quad 2x - 3 \leq -1 && \text{Property 9} \\ x - \frac{3}{2} \geq \frac{1}{2} \quad &\text{or} \quad x - \frac{3}{2} \leq -\frac{1}{2} && \text{Divide by 2.} \\ x \geq 2 \quad &\text{or} \quad x \leq 1 && \text{Add } \tfrac{3}{2}. \end{aligned}$$

The solution set is $(-\infty, 1] \cup [2, \infty)$ (Figure 1.4b). ■

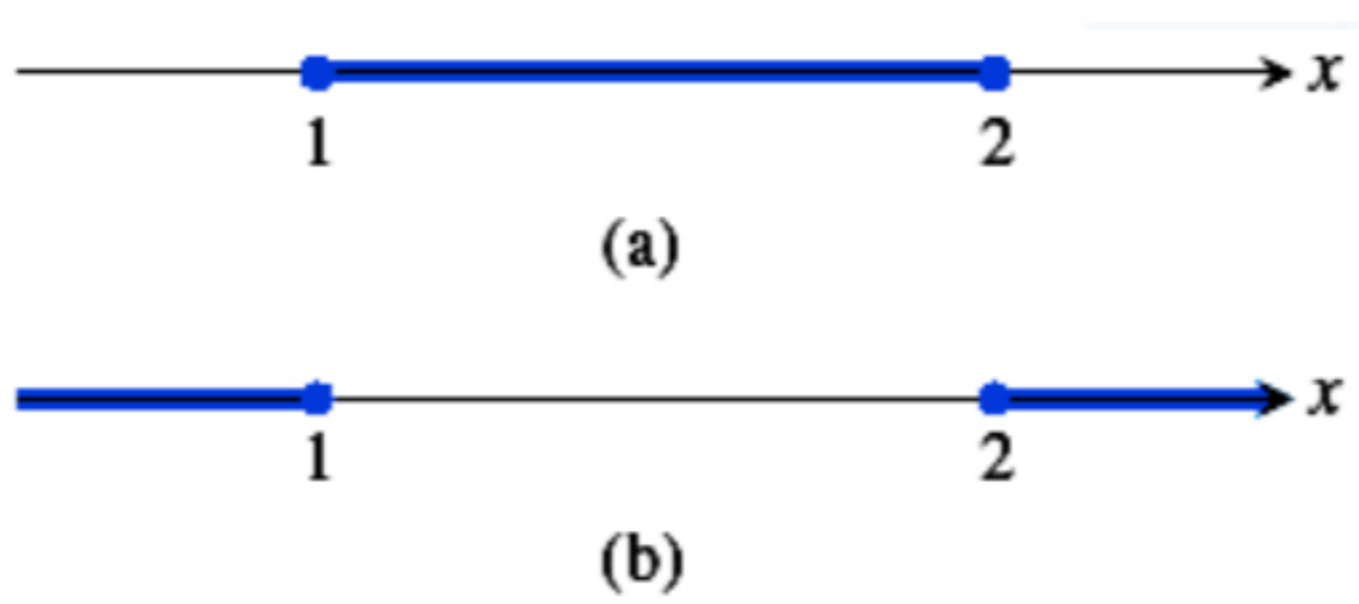

FIGURE 1.4 The solution sets (a) [1, 2] and (b) $(-\infty, 1] \cup [2, \infty)$ in Example 6.

EXERCISES 1.1

Inequalities

3. If $2 < x < 6$, which of the following statements about x are necessarily true, and which are not necessarily true?

   a. $0 < x < 4$
   b. $0 < x - 2 < 4$
   c. $1 < \frac{x}{2} < 3$
   d. $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$
   e. $1 < \frac{6}{x} < 3$
   f. $|x - 4| < 2$
   g. $-6 < -x < 2$
   h. $-6 < -x < -2$

4. If $-1 < y - 5 < 1$, which of the following statements about y are necessarily true, and which are not necessarily true?

   a. $4 < y < 6$
   b. $-6 < y < -4$
   c. $y > 4$
   d. $y < 6$
   e. $0 < y - 4 < 2$
   f. $2 < \frac{y}{2} < 3$
   g. $\frac{1}{6} < \frac{1}{y} < \frac{1}{4}$
   h. $|y - 5| < 1$

In Exercises 5–12, solve the inequalities and show the solution sets on the real line.

5. $-2x > 4$
6. $8 - 3x \geq 5$
7. $5x - 3 \leq 7 - 3x$
8. $3(2 - x) > 2(3 + x)$
9. $2x - \frac{1}{2} \geq 7x + \frac{7}{6}$
10. $\frac{6 - x}{4} < \frac{3x - 4}{2}$
11. $\frac{4}{5}(x - 2) < \frac{1}{3}(x - 6)$
12. $-\frac{x + 5}{2} \leq \frac{12 + 3x}{4}$

Absolute Value

Solve the equations in Exercises 13–18.

13. $|y| = 3$
14. $|y - 3| = 7$
15. $|2t + 5| = 4$
16. $|1 - t| = 1$
17. $|8 - 3s| = \frac{9}{2}$
18. $\left|\frac{s}{2} - 1\right| = 1$

Solve the inequalities in Exercises 19–34, expressing the solution sets as intervals or unions of intervals. Also, show each solution set on the real line.

19. $|x| < 2$
20. $|x| \leq 2$
21. $|t - 1| \leq 3$
22. $|t + 2| < 1$
23. $|3y - 7| < 4$
24. $|2y + 5| < 1$
25. $\left|\frac{z}{5} - 1\right| \leq 1$
26. $\left|\frac{3}{2}z - 1\right| \leq 2$
27. $\left|3 - \frac{1}{x}\right| < \frac{1}{2}$
28. $\left|\frac{2}{x} - 4\right| < 3$
29. $|2s| \geq 4$
30. $|s + 3| \geq \frac{1}{2}$
31. $|1 - x| > 1$
32. $|2 - 3x| > 5$
33. $\left|\frac{r + 1}{2}\right| \geq 1$
34. $\left|\frac{3r}{5} - 1\right| > \frac{2}{5}$

Quadratic Inequalities

Solve the inequalities in Exercises 35–42. Express the solution sets as intervals or unions of intervals and show them on the real line. Use the result $\sqrt{a^2} = |a|$ as appropriate.

35. $x^2 < 2$
36. $4 \leq x^2$
37. $4 < x^2 < 9$
38. $\frac{1}{9} < x^2 < \frac{1}{4}$
39. $(x - 1)^2 < 4$
40. $(x + 3)^2 < 2$
41. $x^2 - x < 0$
42. $x^2 - x - 2 \geq 0$