

## Transformation

### Transformation For Continuous Case

Def) let  $X$  be continuous r.v. with P.d.f.  $f(x)$

Then  $Y = h(x)$  has density fun. Given as:-

$$g(y) = f(x = w(y)) \cdot |J|$$

Where  $J$  is from Jacobin and absolute value

$$\text{of } J = |J| = \left| \frac{dx}{dy} \right|$$

Ex) if  $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{o.w} \end{cases}$

Find P.d.f of  $Y$  when 1)  $Y = -4X + 3$   
2)  $Y = 8X^3$

Sol)

$$g(y) = f(x = w(y)) \cdot |J|$$

↓?      ↓?

1)  $Y = -4X + 3 \rightarrow [+4X = 3 - Y] \div 4$

$$\rightarrow X = \frac{3 - Y}{4}, \quad f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{o.w} \end{cases}$$

$$0 < x < 1 \rightarrow Y = -4x + 3$$

$$\begin{aligned} \text{if } x=0 &\rightarrow Y = -4(0) + 3 \\ &Y = 3 \end{aligned}$$

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$$x=1 \rightarrow y = -4x + 3 \Rightarrow y = -1$$

$$\therefore -1 < y < 3$$

$$x = \frac{3-y}{4} \Rightarrow \therefore x = \frac{3}{4} - \frac{y}{4}, \quad |J| = \left| \frac{dx}{dy} \right|$$

$$|J| = \left| -\frac{1}{4} \right| = \frac{1}{4}$$

$$\therefore g(y) = f(x=w(y)) \cdot |J| \\ = 2 \left( \frac{3-y}{4} \right) \cdot \left( \frac{1}{4} \right)$$

from  $\rightarrow f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{o.w} \end{cases}$

$$\therefore g(y) = \begin{cases} (3-y)/8, & -1 < y < 3 \\ 0, & \text{o.w} \end{cases}$$

2) since  $y = 8x^3$ ,  $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{o.w} \end{cases}$

$$x^3 = \frac{y}{8} \rightarrow x = \sqrt[3]{\frac{y}{8}} = \frac{y^{1/3}}{2} \leftarrow \text{الجزء الكلي}$$

$$|J| = \frac{1}{6} y^{-2/3}$$

$$x=0 \rightarrow y = 8(0)^3 = 0 \\ x=1 \rightarrow y = 8(1)^3 = 8$$

$$\therefore 0 < y < 8$$

$$\therefore g(y) = f(x=w(y)) \cdot |J|$$

$$\rightarrow g(y) = \left\{ \frac{1}{6} y^{-1/3}, \quad 0 < y < 8 \right\}$$

## ∴ Transformation ∴

Ex<sub>2</sub>) if  $X \sim N(0,1)$ . Find  $g(y)$  by Transformation method if  $Y = |X|$ .

Sol)

$$X \sim N(0,1) \Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, & -\infty < x < \infty \\ 0, & \text{o.w} \end{cases}$$

$$g(y) = f(x=w(y)) \cdot |J|$$

$$\rightarrow Y = |X| \rightarrow \text{by Transformation } x = \pm y$$

$$\rightarrow |J| = \left| \frac{dx}{dy} \right| = |\pm 1| = 1$$

$$\rightarrow g(y) = f(x=+y) \cdot |+1| + f(x=-y) \cdot |-1|$$

$$\Rightarrow f(x=+y) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, & 0 < y < \infty \\ 0, & \text{o.w} \end{cases}$$

$$f(x=-y) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, & 0 < y < \infty \\ 0, & \text{o.w} \end{cases}$$

$$\therefore -\infty < x < \infty \rightarrow y = |x| \begin{cases} \rightarrow D_F = \mathbb{R} \\ \rightarrow R_F = \{y: 0 < y < \infty\} \\ \quad (0, +\infty) \end{cases} \quad ??$$

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∴ Transformation ∴

$$g(y) = \begin{cases} \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}, & 0 < y < \infty \\ 0, & \text{o.w} \end{cases}$$

EX3) if  $X \sim \text{Beta}(a, b)$ , Find  $g(y)$  by using Transformation method if  $y = 1+x$ .

SOL)  $y = 1+x \rightarrow x = y-1$

$$|J| = \left| \frac{dx}{dy} \right| = |1| = 1$$

$$\therefore X \sim \text{Beta}(a, b) \Rightarrow f(x) = \begin{cases} \frac{1}{\beta(a, b)} x^{a-1} (1-x)^{b-1}, & 0 \leq x \leq 1 \\ 0, & \text{o.w} \end{cases}$$

$$\Rightarrow g(y) = f(x=w(y)) |J|$$

$$= \frac{1}{\beta(a, b)} \cdot (y-1)^{a-1} \cdot (1-(y-1))^{b-1} \cdot |1|$$

$$\Rightarrow g(y) = \begin{cases} \frac{1}{\beta(a, b)} \cdot (y-1)^{a-1} \cdot (2-y)^{b-1}, & 1 \leq y \leq 2 \\ 0, & \text{o.w} \end{cases}$$

H.W) if  $f(x) = \begin{cases} 2x e^{-x^2}, & x > 0 \\ 0, & \text{o.w} \end{cases}$

Find P.d.f of  $y$  if  $y = x^2$ .