((chapter two))

الاحصاءات المرتبة Order statistics

The Oder statistics of aset of random variables of $X_1, X_2, \dots, X_n$ are the same random variables arranged in increasing order.

Denote by

$Y_1 = X_{(1)}$ = smallest of $X_1, X_2, \dots, X_n = \min(X_i)$

$Y_2 = X_{(2)} = 2^{nd}$ smallest of $X_1, X_2, \dots, X_n$

$\vdots$

$Y_n = X_{(n)}$ = largest of $X_1, X_2, \dots X_n = \max(X_i)$

Note:-

* Even if X_i's are independent, Y_i's $= X_{(i)}$'s can not b independent since

$$X_{(1)} \leqslant X_{(2)} \leqslant \dots \leqslant X_{(n)}$$

* Distribution of X_i's and Y_i's $= X_{(i)}$'s are not the same

∴ Order Statistics ∴

Distribution of the k-th order statistic

$$Y_{(k)} = X_{(k)}$$

Suppose $X_1, X_2, \dots, X_n$ are i.i.d random variables with common distribution function $F_x(x)$ and common density function $f_x(x)$.

– The density function of $X_{(k)}$ or $Y_{(k)}$ is:-

$$g(y_k) = \frac{n!}{(k-1)!\,(n-k)!} \, (\underbrace{F_x(y_k)}_{c.d.f})^{k-1} \cdot (1 - F_x(y_k))^{n-k} \cdot f_x(y_k).$$

أغير في دالة التوزيع
متغير x ع
فيرسل لـ x_k
y_k

P.d.f of the smallest order statistics

– if $X_1, X_2, \dots, X_n$ be a r.s of size n from a Population with continuous p.d.f $f(x)$, then the p.d.f of the Smallest order statistics $X_{(1)}$ is given as:-

$$g(y_1) = n\,(1 - \underbrace{F_x(y_1)}_{\text{dist. fun.}})^{n-1} \cdot \underbrace{f_x(y_1)}_{\text{density fun.}}$$

$$F(y_k) = \int_{\text{أصغر قيمة } x}^{y_k} f(x)\,dx$$

∴ Order Statistics ∴

Example:-

let $X_i \sim Exp(1)$; $i=1,\dots,5$ and Y_i; $i=1,\dots,5$ be OS, find $g(y)$ where $y=\min(X_i)$

Sol

∵ Find $g(y) \longrightarrow y=\min(X_i)$

∴ smallest order statistic

$\boxed{n=5}$

$$g(y_1)=n\left(1-F_x(y_1)\right)^{n-1}.f_x(y_1)$$

بالتعويض

$$g(y_1)=5\left(1-F_x(y_1)\right)^{4}.f_x(y_1)$$

$\frac{1}{\theta}e^{-\frac{x}{\theta}}$ $\theta=1$

$$\because X_i \sim Exp(1) \rightarrow f(x)=\begin{cases} e^{-x} & , x>0 \\ 0 & , o.w. \end{cases}$$

$$\Rightarrow f_x(y_1)=e^{-y_1} \quad , y_1>0$$

$$F(x)=\int_0^x f(u)\,du=\left[\int_0^x e^{-u}.du\right]*\left(\frac{-1}{-1}\right) \text{ ?}$$

$$=-1.\int_0^x -e^{-u}du=-1\left[e^{-u}\right]_0^x=-\left[e^{-x}-1\right]$$

$$F(x)=1-e^{-x} \quad , F_x(y_1)=1-e^{-y_1}$$

$$\Rightarrow g(y_1)=5\left(1-F_x(y_1)\right)^4.f_x(y_1)$$

$$=5\left(1-(1-e^{-y_1})\right)^4.e^{-y_1}=\begin{cases} 5(e^{-y_1})^5 & , y_1>0 \\ 0 & , o.w. \end{cases}$$