

## $\therefore$ Order Statistics $\therefore$

P.d.f of the largest order statistics:

- if  $X_1, X_2, \dots, X_n$  be a r.s. of size  $n$  from a population with continuous p.d.f  $f(x)$ , then the p.d.f of the largest order statistic  $Y_n$  is given as:

$$g(y_n) = n (F_x(y_n))^{n-1} \cdot f_x(y_n)$$

Example

- Suppose  $X_1, X_2$  are i.i.d (identically independent distributed) Uniform  $(0, 1)$  random variables, find the density function of  $Y_2$ .

Sol.

$\therefore g(y_2)$  and  $n=2$

$\therefore$  p.d.f of the largest O.S.

$$\rightarrow g(y_n) = n (F_x(y_n))^{n-1} \cdot f_x(y_n)$$

$$\therefore n=2 \rightarrow g(y_2) = 2 (F_x(y_2))^{2-1} \cdot f_x(y_2)$$

$$\Rightarrow X \sim \text{Uni}(0, 1)$$

$$\therefore f(x) = \begin{cases} \frac{1}{1-\theta} & , \theta < x < 1 \\ 0 & , \text{o.w.} \end{cases}$$

$$\rightarrow f_x(y_2) = \frac{1}{1-\theta} \quad , \quad F(x) = \int_0^x f(u) du$$

## ∴ Order Statistics ∴

$$F(x) = \int_{\theta}^x f(u) du = \int_{\theta}^x \frac{1}{1-\theta} du = \frac{1}{1-\theta} \int_{\theta}^x du$$

$$F(x) = \frac{1}{1-\theta} [u]_{\theta}^x = \frac{1}{1-\theta} \cdot (x - \theta) \quad , \quad \theta < x < 1$$

$$F_x(y_2) = \frac{y_2 - \theta}{1 - \theta} \quad , \quad \theta < y_2 < 1$$

$$\therefore g(y_2) = 2 (F_x(y_2)) \cdot f_x(y_2) = 2 \left[ \frac{y_2 - \theta}{1 - \theta} \right] \frac{1}{1 - \theta}$$

$$g(y_2) = \left\{ \begin{array}{ll} \frac{2(y_2 - \theta)}{(1 - \theta)^2} & , \quad \theta < y_2 < 1 \\ 0 & , \quad \text{o.w.} \end{array} \right\}$$

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Example:

$$\text{let } f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{o.w.} \end{cases}$$

and  $y_i, i=1, 2, 3$  are Os, find  $g(y_2)$

Sol:

$n=3$ ,  $g(y_2)$  in general

$$g(y_k) = \frac{n!}{(k-1)!(n-k)!} \cdot (F_x(y_k))^{k-1} \cdot (1 - F_x(y_k))^{n-k} \cdot f_x(y_k)$$

$$n=3, k=2$$

$$\therefore g(y_2) = \frac{3!}{(2-1)!(3-2)!} \cdot (F_x(y_2))^{2-1} \cdot (1 - F_x(y_2))^{3-2} \cdot f_x(y_2)$$

$$\therefore f(x) = 1 \Rightarrow f_x(y_2) = 1$$

$$F(x) = \int_0^x f(u) du = \int_0^x 1 du = u \Big|_0^x = x - 0 = x$$

$$F_x(y_2) = y_2 \quad \text{then } g(y_2) = 6(y_2)(1-y_2) \cdot 1$$

$$\Rightarrow g(y_2) = \begin{cases} 6(y_2)(1-y_2), & 0 < y_2 < 1 \\ 0, & \text{o.w.} \end{cases}$$

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"Order Statistics"

Example: let  $F(x) = \begin{cases} 2(1-x) & , 0 \leq x < 1 \\ 0 & , \text{o.w.} \end{cases}$

and  $y_i, i=1, \dots, 5$  be OS, find  $g(y)$  where  $y = \max(x_i)$

Sol  $n=5$ , Find  $y = \max(x_i) \rightarrow g(y_5)$

$$\Rightarrow g(y_n) = n (F_x(y_n))^{n-1} \cdot f_x(y_n)$$

$$g(y_5) = 5 (F_x(y_5))^4 \cdot f_x(y_5)$$

$$\rightarrow F(x) = 2(1-x) \rightarrow f_x(y_5) = 2(1-y_5)$$

$$F(x) = \int_0^x f(u) du = \int_0^x 2(1-u) du = 2 \int_0^x (1-u) du$$

$$F(x) = 2 \left[ u - \frac{u^2}{2} \right]_0^x = 2 \left[ x - \frac{x^2}{2} \right]$$

$$F_x(y_5) = 2 \left( y_5 - \frac{y_5^2}{2} \right) \quad , 0 \leq y_5 < 1$$

$$g(y_5) = 5 \left( 2y_5 - y_5^2 \right)^4 \cdot 2(1-y_5)$$