الفصل الثاني

- The joint distributions for The order statistics.

For any $i<j$ The joint OS dist. Between Y_i and Y_j is given as

$$g(Y_i,Y_j) = \frac{n!}{(i-1)!\,(j-i-1)!\,(n-j)!} \cdot [F_x(Y_i)]^{(i-1)} \cdot$$

$$[F_x(Y_j) - F_x(Y_i)]^{(j-i-1)} \cdot [1 - F_x(Y_j)]^{n-j} \cdot f_x(Y_i) \cdot f_x(Y_j)$$

$$a < Y_i , Y_j < b$$

Ex

let $X_i \sim Exp(1)$, $i=1,\dots,5$, and Y_i; $i=1,\dots,5$ be OS

find $g(Y_2, Y_4)$.

Sol/ we have $n=5$, $i=2$, $j=4$

$$g(Y_2,Y_4) = \frac{5!}{(2-1)!\,(4-2-1)!\,(5-4)!} \cdot [F_x(Y_1)]^{2-1}$$

$$\cdot [F_x(Y_4) - F_x(Y_2)]^{(4-2-1)} \cdot [1 - F_x(Y_4)]^{5-4} \cdot f_x(Y_2) \cdot f_x(Y_4)$$

$$= 120\, [F_x(Y_1)] \cdot [F_x(Y_4) - F_x(Y_2)] \cdot [1 - F_x(Y_4)]$$

$$\cdot f_x(Y_1) \cdot f_x(Y_4).$$

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②

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & o.w \end{cases}$$

$f_x(y_2) = e^{-y_2}$ and $f_x(y_4) = e^{-y_4}$

$F(x) = \int_0^x f(u)\,du \Rightarrow \int_0^x e^{-u}\,du \ast \left(\frac{-1}{-1}\right)$

$-1\int_0^x -e^{-u}\,du \Rightarrow -1\left[e^{-u}\right]_0^x \Rightarrow -\left[e^{-x}-1\right]$

$F(x) = (1 - e^{-x})$

$F_x(y_2) = (1 - e^{-y_2})$, $F_x(y_4) = (1 - e^{-y_4})$

$= 120\,[1 - e^{-y_2}][(1 - e^{-y_4}) - (1 - e^{-y_2})][1 - (1 - e^{-y_4})]\,e^{-y_2}\,e^{-y_4}$

$$g(y_2, y_4) = \begin{cases} 120\, e^{-2y_4}\, e^{-y_2}\,[1 - e^{-y_2}][e^{-y_2} - e^{-y_4}] & 0 < y_2, y_4 < \\ 0 & o.w \end{cases}$$

EX/ let $f(x) = \begin{cases} 1 & 0 \le x < 1 \\ 0 & o.w \end{cases}$

and Y_i ; $i = 1, 2, 3$ are OS find $g(y_1, y_3)$ and $g(y_3$

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Sol/ $n = 3$, $i = 1$ and $j = 3$

$$g(y_1, y_3) = \frac{3!}{(1-1)!\,(3-1-1)!\,(3-3)!}\left[F_x(y_1)\right]^{(1-1)}$$

$$\cdot\left[F_x(y_3) - F_x(y_1)\right]^{3-1-1} \cdot \left[1 - F_x(y_3)\right]^{3-1} \cdot f_x(y_1) \cdot f_x(y_3)$$

$$g(y_1, y_3) = 6\left[F_x(y_3) - F_x(y_1)\right] f_x(y_1) \cdot f_x(y_3)$$

$f(x) = 1$ $\quad f_x(y_1) = 1$ and $f_x(y_3) = 1$

$$F(x) = \int_0^x f(u)\,du \Rightarrow \int_0^x du \Rightarrow u\Big]_0^x = x$$

$F_x(y_1) = y_1$ and $F_x(y_3) = y_3$

$$g(y_1, y_3) = 6(y_3 - y_1) \cdot (1) \cdot (1)$$

$$g(y_1, y_3) = \begin{cases} 6(y_3 - y_1) & 0 < y_1, y_3 < 1 \\ 0 & \text{o.w} \end{cases}$$

(2) $g(y_3) = n\left[F_x(y_3)\right]^{3-1} \cdot f_x(y_3)$

$$g(y_3) = \begin{cases} 3y_3^2 & 0 < y_3 < 1 \\ 0 & \text{o.w} \end{cases}$$

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