"Chapter three"

التقدير "Estimation"

1_ Unbiasedness :_ عدم التحيز

let $X_1, X_2, \dots, X_n$ be arandom sample from apopulation having pdf $f(x;\theta)$ with unknown Parameter θ. let $\hat{\theta}$ be an estimator of θ, Therefore $\hat{\theta}$ is said to be Unbiased estimator of θ, iff $E(\hat{\theta}) = \theta$, Otherwise, the bias of an estimator $\hat{\theta}$ can be defined as

التقدير يكون unbaised في حال

$\theta = E(\hat{\theta})$ أي أن توقع الـ $\hat{\theta}$ حقيقة

التحيز

مقدار التحيز

$$bias(\theta) = E(\hat{\theta}) - \theta$$

* يكون التقدير غير متحيز في حالة كانت قيمة $b = 0$، وبالتالي يصبح

* Properties * وتساوي $0 = E(\hat{\theta}) - \theta \Rightarrow E(\hat{\theta}) = \theta$ غير متحيز للمعلمة θ

1) $E(k) = k$, k is a constant.
2) $E(kx) = kE(x)$
3) $E(kx \mp cy) = kE(x) \mp cE(y)$ where k and c are constants.
4) $v(k) = 0$
5) $v(kx) = k^2 v(x)$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

*** Properties of a Good Estimator :_**

An estimator $\hat{\theta}$ is said unbiased if its expected value equals the value of Parameter θ being estimated, That is

1) if $E(\hat{\theta}) = \theta$, $\hat{\theta}$ is Unbiased
2) if $E(\hat{\theta}) \neq \theta$, $\hat{\theta}$ is biased

Estimation

Ex) let $X_i \sim Exp(\theta)$, $i=1,\dots,n$, is $\hat{\theta}=\bar{x}$ is an Unbiased for θ?

Solution)

$$E(\hat{\theta}) = E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(x_i)$$

since $X_i \sim Exp(\theta)$ $\therefore E(X_i)=\theta$

$$\rightarrow E(\hat{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\theta = \frac{n\theta}{n} = \theta \qquad \sum_{i=1}^{n}\theta = \theta+\theta+\theta+\theta+\theta\dots = n\theta$$

$\rightarrow \therefore E(\hat{\theta})=\theta$, Then $\hat{\theta}$ is Unbiased for the Parameter θ. يعني جمع θ من n المرات

Ex) let $X \sim Gam(\theta,\beta)$ is $\hat{\theta}=\bar{x}$ unbiased for θ?

Solution)

$$E(\hat{\theta}) = E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\left(\sum_{i=1}^{n} E(x_i)\right)$$

Gam الصيغة للتوقع
$Gam(\alpha,\beta) \rightarrow E: \alpha\beta$ $\rightarrow \because X \sim Gam(\theta,\beta)$ $\therefore E(X)=\theta\beta$

$$\rightarrow E(\hat{\theta}) = \left[\frac{1}{n}\sum_{i=1}^{n}\theta\beta\right] = \frac{n\theta\beta}{n} = \theta\beta$$

$$\therefore E(\hat{\theta}) \neq \theta$$

So $\hat{\theta}=\bar{x}$ is biased estimator for θ.

Note) 3- The unbiased estimator for above Ex. will be

$$\hat{\theta} = \frac{\bar{x}}{\beta}$$

H.W) let $X_i \sim U(\theta,\theta+1)$, $i=1,2,\dots,n$

show that $\hat{\theta} = \bar{x}-\frac{1}{2}$ is unbiased estimator for θ?