Chapter Three

2- Mean Square error (MSE) متوسط مربع الخطأ

If $\hat{\theta}$ is an estimator of the parameter θ Then the mean square error can be defined by

$$MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2$$

$$= E(\hat{\theta}^2 - 2\hat{\theta}\theta + \theta^2)$$

$$= E(\hat{\theta}^2) - 2E(\hat{\theta})\underbrace{E(\theta)}_{\text{ثابت}} + E(\theta^2)$$

$$= E(\hat{\theta}^2) - 2E(\hat{\theta})\cdot\theta + \theta^2 \quad (\theta \text{ ثابت})$$

بإضافة العدد $\left[-[E(\hat{\theta})]^2 + [E(\hat{\theta})]^2\right]$ الى المعادلة السابقة

$$= E(\hat{\theta}^2) - [E(\hat{\theta})]^2 + [E(\hat{\theta})]^2 - 2\theta E(\hat{\theta}) + \theta^2$$

$$= \left[E(\hat{\theta}^2) - (E(\hat{\theta}))^2\right] + [E(\hat{\theta})]^2 - 2\theta E(\hat{\theta}) + \theta^2$$

$$= Var(\hat{\theta}) + (E(\hat{\theta}) - \theta)^2$$

$$= Var(\hat{\theta}) + (bais(\hat{\theta}))^2$$

$$\therefore MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2$$

$$= Var(\hat{\theta}) + bias^2(\hat{\theta})$$

ملاحظة /

اذا كانت $\hat{\theta}$ غير متحيزة للمعلمة θ ، بحيث $E(\hat{\theta}) = \theta$ هذا

يعني ان $bias(\theta) = 0$ ، اي ان

$$MSE(\hat{\theta}) = Var(\hat{\theta})$$

- If $MSE(\hat{\theta})$ is smaller, $\hat{\theta}$ is a better estimater of θ.

- For two estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ of θ, if $MSE(\hat{\theta}_1) < MSE(\hat{\theta}_2)$, $\hat{\theta}_1$ is better estimator of θ.

Ex/ Let we have two estimators for the parameter θ which they are $\hat{\theta}_1$ and $\hat{\theta}_2$ where $MSE(\hat{\theta}_1) = 0.758$ and $MSE(\hat{\theta}_2) = 0.427$ which estimator is better?

Sol/

Since $MSE(\hat{\theta}_1) = 0.758 > MSE(\hat{\theta}_2) = 0.427$

$\Rightarrow \hat{\theta}_2$ is better estimator for θ.