Efficient for more Than one estimator.

Defi- If we have $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbaised estimator for θ Then efficient estimator is one with less Variance

i.e.

$$\text{eff} = \frac{V(\hat{\theta}_1)}{V(\hat{\theta}_2)}$$

اذا ثبت حالة عدم التحيز للمقدرين $\hat{\theta}_1$ و $\hat{\theta}_2$: ② نذهب إلى الكفاءة

اذا كانت النسبة اقل من (1) فأن المقدر الاول هو الاكفأ والعكس صحيح.

Ex/

let X_i ; $i = 1, 2, 3$ be indep. r.v.s with mean (توقع) θ and variance (تباين) σ^2 Find The efficient estimator between

$$\hat{\theta}_1 = \frac{(X_1 + 2X_2 + 3X_3)}{6} \quad \text{and} \quad \hat{\theta}_2 = \bar{X}$$

Sol/

① we See if They are unbiased estimator by

$$E(\hat{\theta}_1) = E\left(\frac{X_1 + 2X_2 + 3X_3}{6}\right) = \frac{E(X_1) + 2E(X_2) + 3E(X_3)}{6}$$

كل توقع نعوضه θ حسب السؤال لأنه معطى ان التوقع هو θ

$$= \frac{\theta + 2\theta + 3\theta}{6} = \frac{\cancel{6}\theta}{\cancel{6}} = \theta$$

$E(\hat{\theta}_2) = E(\bar{X}) = \theta$ So They are unbiased estimator

$$E\left(\frac{\sum X_i}{n}\right) = \frac{1}{n} \sum E(X_i) = \frac{1}{n} \sum \theta = \frac{1}{\cancel{n}} \cdot \cancel{n}\theta = \theta$$

② Now we have to find $V(\hat{\theta}_1)$ and $V(\hat{\theta}_2)$ and The most efficient one is The one with The Smallest Variance.

$$V(\hat{\theta}_1) = V\left[\left(\frac{X_1 + 2X_2 + 3X_3}{6}\right)\right] =$$

Since X_i, $i = 1, 2, 3$ be indep. r.v.s Then

$$V(\hat{\theta}_1) = \frac{1}{36}\left[V(X_1) + 4V(X_2) + 9V(X_3)\right]$$

$V(6) = 6^2 = 36$

مساوي لكل Variable

$$= \frac{\sigma^2 + 4\sigma^2 + 9\sigma^2}{36} = \frac{7}{18}\sigma^2$$

هو σ^2 مثل كل var

نضع σ^2

$$V\left[\frac{\sum x_i}{n}\right] = \frac{1}{n^2} V \sum x_i = \frac{1}{n^2} \sum V(x_i) = \frac{1}{n^2} \sum \sigma^2 = \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n} = \frac{\sigma^2}{3}$$

and $V(\hat{\theta}_2) = V(\bar{X}) = \dfrac{\sigma^2}{3}$

$V(\hat{\theta}_1) > V(\hat{\theta}_2)$ Then $\hat{\theta}_2 = \bar{X}$ is The efficient estimator for θ.