8. Uniqueness

To find The uniqueness estimator for The parameter θ we found The following steps.

1. Find $T(x)$ The Comp. S.S. for θ
2. Find The expected value of T.
3. find The unbiased estimator $\hat{\theta}$ as a function of T

Then $\hat{\theta}$ is The best unique estimator for θ.

Ex1/ let $x \sim Exp(\theta), i=1,2,\dots,n$ is $\bar{x}$ unique estimator for θ?

Sol/

1. by exp. family of distribution

$$f(x) = \exp[\ln f(x,\theta)] = \exp\left[\ln\left(\frac{1}{\theta} e^{-\frac{x}{\theta}}\right)\right]$$

$= \exp\left[\ln(1) - \ln(\theta) - \frac{x}{\theta}\right]$

So $a(\theta) = \ln(1) - \ln(\theta)$, $b(x) = 0$, $C(\theta) = -\frac{1}{\theta}$

and $d(x) = x$

$T = \sum_{i=1}^{n} d(x_i) = \sum_{i=1}^{n} x_i$ is a Comp. S.S. for θ.

(2) $E(T) = E(\sum x_i) = \sum E(x_i) = \sum \theta = n\theta$

(3) To find The unbiased estimator ~~for~~ as a function of T we have

$$\hat{\theta} = \frac{T}{n} = \frac{\sum x_i}{n} = \bar{x}$$ is The unbiased estimator for θ.

So. $\hat{\theta} = \bar{x}$ is The best unique estimator for θ.

H.w/ let $X_i \sim Ber(\theta)$ $i=1,2,\dots,n$ is $\bar{x}$ unique estimator for θ?