Chapter three

The natural logarithm and exponential Function and L' Hopital Rule

1- The natural logarithmic Function دالة اللوغارتم الطبيعي

The natural logarithm denoted by $\mathbf{ln}(\boldsymbol{x})$

Properties of $\ln(x)$:

1- $D_f = \{x: x > 0\} = R^+/\{0\}$

2- $R_f = \{-\infty < y > 0\} = R\}$

3-$\ln(1) = 0$

4-$\ln(e) = 1$ where e=2.7182

5- $\ln(a.b) = \ln(a) + \ln(b)$

6-$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

7-$\ln a^r = rln(a)$ $r \in R$, $a \in R^+$

8- $\ln\left(\frac{1}{x}\right) = -\ln x$

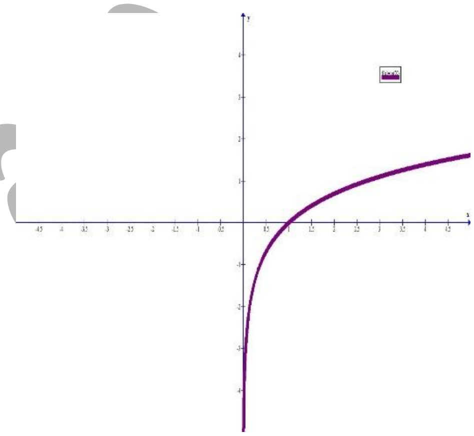
9-$\ln(x) > 0$ if $x > 0$

10- $\ln(x) < 0$ if $x < 0$

11- $\lim_{x \to 0} \ln x = -\infty$

12- $\lim_{x \to \infty} \ln x = \infty$

The graph of $\ln x$



1- $y = \ln x$

$\Rightarrow \quad D_y = R^+ \quad , R_y = R$

2- $\ln 1 = 0$ (0,1) نقطة التقاطع

3- $\lim_{x \to 0} \ln x = -\infty$

4- $\lim_{x \to \infty} \ln x = \infty$

Derivative of $\ln x$

$$y = \ln x \ \rightarrow \ y' = \frac{1}{x} \cdot 1$$

Or

$y = \ln u$ where u is function

$$y' = \frac{1}{u} \cdot du$$

Example:

1- let $y = \ln x^2 \Rightarrow y' = \frac{1}{x^2}\,(2x) = \frac{2}{x}$

2- let $y = \ln(\sin x) \Rightarrow y' = \frac{1}{\sin x}\,(\cos x) = \cot x$

3- let $y = [\ln(x^2+2)]^5 \Rightarrow y' = 5\,[\ln(x^2+2)]^4\,(\frac{1}{x^2+2})(2x)$

4- let $y = \ln(x^2 + 3x + 1)$

$$y' = \frac{1}{(x^2 + 3x + 1)}(2x + 3)$$

5- $y = ln\left(\frac{x^2+1}{x}\right)$

$$ln\left(\frac{a}{b}\right) = ln(a) - ln(b)$$

$$\therefore y = ln(x^2 + 1) - ln(x)$$

$$y' = \frac{2x}{x^2 + 1} - \frac{1}{x}$$

2- The exponential Function الدالة الأسية

هي معكوس دالة اللوغاريتم الطبيعي ويرمز
لها بالرمز exp or e^x

<u>Properties of $\exp(x)$</u>

1- $y = e^x \Rightarrow \quad x = \ln y$

2- $D_y = R \quad , R_y = R^+$

3- $e^0 = 1$

4- $e^a . e^b = e^{a+b}$

5- $\frac{e^a}{e^b} = e^{a-b}$

6- $(e^a)^r = e^{ar}$

7- $e^{-a} = \frac{1}{e^{a}}$

8- $\ln e^{x} = x \qquad \forall\, x > 0$

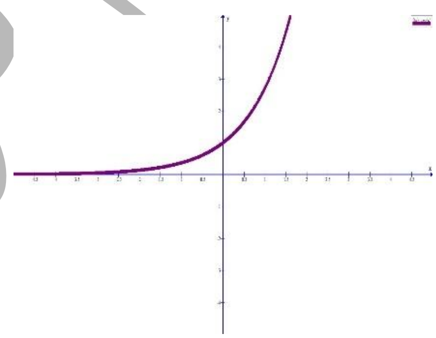
9- $\lim_{x \to \infty} e^{x} = e^{\infty} = \infty$

10- $\lim_{x \to -\infty} e^{x} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

The graph of e^{x}:

1- $y = e^{x}$

$$\Rightarrow \qquad D_y = R \qquad , R_y = R^{+}$$



2- $e^{0} = 1$ (1,0) نقطة التقاطع

3- $\lim_{x \to \infty} e^{x} = e^{\infty} = \infty$

4- $\lim_{x \to -\infty} e^{x} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

Derivative of e^{x}:

Let $y = e^{x} \quad \Rightarrow y' = e^{x} \,.\, 1$

Or

If $y = e^{u}$ where u is a function, then

$$y' = e^{u} \,.\, du$$

Example:

Find y' of the following function

1- $y = e^{\tan x} \Rightarrow y' = e^{\tan x}.(sec^2\, x).1$

2- $y = e^{x^2+\sin x} \Rightarrow y' = e^{x^2+sinx}.(2x + \cos x)$

3-$y = \sin(e^{x^2}) \Rightarrow y' = \cos(e^{x^2})[e^{x^2}(2x)]$

$$= (2x)(e^{x^2}).[\cos(e^{x^2})]$$

Example:

Find $\grave{y}$ y'' y''' of $y = e^{x+1}$ and $y'''(-1)$

Sol:

$\grave{y} = e^{x+1}.(1) = e^{x+1}$

$y'' = e^{x+1}(1) = e^{x+1}$

$y''' = e^{x+1}(1) = e^{x+1}, y'''(-1) = e^{-1+1} = e^0 = 1$

H.w

find $\grave{y}$ for each the following function

1- $y = \sin(\ln x)$

2- $y = \cos(e^x)$

3- $y = \frac{e^x}{\sin x + 1}$

4- $y = \frac{\sin x + \sec x}{\ln x}$

5- $y = e^{\tan x + \sqrt{x}}$

6- $y = e^{\sin x + \ln x}$

7-$y = ln\left(\sqrt{x^2 + 1}\right)$

8-$y = ln\left(e^{x^2} + 1\right)$

9-$y = ln(x^2 sinx)$

10- $y = ln\left(ln(x)\right)$

11- $y = \frac{e^x}{x}$