3- The L´Hopital Rule قاعدة لوبيتال

Theorem:

If $f(x)$, $g(x)$, are two differential function on open interval (c,d) s.t $a\epsilon(c,d)$ and $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{\grave{f}(x)}{\grave{g}(x)}, \quad s.t \quad \grave{g}(x) \neq 0$$

- تستخدم قاعدة لوبيتال عندما يكون ناتج الغاية للدالة الكسرية غير محدد مثلاً $(\frac{0}{0})$ او $(\frac{\infty}{\infty})$. لذلك نطبق قاعدة لوبيتال على مثل هذه الغايات (اي نشتق البسط والمقام كلاً على حده ثم نعوض بالغاية) .

Example:

1- $\lim_{x\to 3}\frac{x^2-9}{x-3}$

Sol/

$$\lim_{x\to 3}\frac{2x}{1} = \lim_{x\to 3} 2x = 6$$

2- $\lim_{x\to 0}\frac{\sin 7x}{x}$

Sol/

$$\lim_{x\to 0}\frac{\sin 7x}{x} = \lim_{x\to 0}\frac{7\ \cos\ 7x}{1} = 7\cos 0 = 7$$

3- $\lim_{x\to\infty} \frac{x}{e^x}$

Sol/

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$$

4- $\lim_{x\to 0^+} \frac{\ln x}{\csc x}$

Sol/

$$\lim_{x\to 0^+} \frac{\ln x}{\csc x} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x} = \lim_{x\to 0^+} -\frac{1}{x}\left[\frac{1}{\frac{1}{\sin x} . \frac{\cos x}{\sin x}}\right]$$

$$= \lim_{x\to 0^+} -\frac{1}{x}\left[\frac{sin^2 x}{\cos x}\right]$$

$$\lim_{x\to 0^+} \frac{sinx}{x}\left[\frac{-\sin x}{\cos x}\right]$$

$$= 1 . 0 = 0$$

<u>H.W</u>

Show that

1- $\lim_{x\to 2} \frac{e^x - e^2}{x-2} = e^2$

2-$\lim_{x\to 0} \frac{x\ e^x}{1-e^x} = -1$

3-$\lim_{x\to 3} \frac{x^2-3x}{x^2-9} = \frac{1}{2}$

4- $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = 0$

5- $\lim\limits_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$

6- $\lim\limits_{x \to 0} \frac{e^{x}\text{-1-x}}{\mathrm{x}^2}$

6- find $\grave{y}$ y'' y''' for each the following function

a. $y = \tan(\mathrm{e^x}) + \cos(\ln \mathrm{x})$
b. $y = csc\ (x^2 + 5).\, e^x$
c. $y = (e^x + 1)\ sin\ (x^2)$
d. $y = \tan\ (lnx).\,(x^2 + 2x)$
e. $y = \frac{tan\ x}{ln\ x} + 10$
f. $y = e^x + tan\ x - sec\ x + sin\ x - ln\ x + cos\ x$
g. $y = sin\ x + cos\ x - tan\ x + cos\ x - sec\ x + csc\ x - e^x + ln\ x$

4- Application of derivatives

تطبيقات على المشتقات

Slope and Tangent line and Normal line

1) Definition:

let $y = f(x)$. Then the slope of curve $y = f(x)$ at any point $p(x, y)$;

$$m = \grave{y} = \grave{f}(x)$$

الميل = المشتقة عند النقطة p

2) Definition:

the equation of the tangent line to the curve $y = f(x)$, which pass through the point $p_0(x_0, y_0)$ is

$$(y - y_0) = m(x - x_0)$$ معادلة المماس

3) Definition:

The equation of Normal line to the curve $y = f(x)$, which pass through the point $p_0(x_0, y_0)$ is

$$(y - y_0) = \frac{-1}{m}(x - x_0) \quad \text{معادلة ميل العمود}$$

ميل العمود =مقلوب الميل عكس الاشارة

Example:

Find the equation of each of the tangent line and the normal line to the curve of the function $y = f(x) = x^4 - x^3 - 2x^2 - 2x - 2$

At the point $(2, -6)$

Sol:

$$\grave{f}(x) = 4x^3 - 3x^2 - 4x - 2$$

Since $m = \grave{f}(x_0)$

$$m = 4(2)^3 - 3(2)^2 - 4(2) - 2$$

$$= 32 - 12 - 8 - 2 = 10$$

Now,

The equation of tangent of line at $(2, -6)$ is

$$(y - y_0) = m(x - x_0)$$

$$(y + 6) = 10(x - 2)$$

$$y - 10x = -6 - 20 \qquad \Rightarrow \qquad y - 10x + 26 = 0$$

To find equations of normal line

$$(\boldsymbol{y}-\boldsymbol{y_0})=\frac{\boldsymbol{-1}}{\boldsymbol{m}}(\boldsymbol{x}-\boldsymbol{x_0})$$

$$\Rightarrow (\boldsymbol{y}+\boldsymbol{6})=\frac{\boldsymbol{-1}}{\boldsymbol{10}}(\boldsymbol{x}-\boldsymbol{2})$$

$$\boldsymbol{y}+\boldsymbol{6}=\frac{\boldsymbol{-1}}{\boldsymbol{10}}\boldsymbol{x}+\frac{\boldsymbol{1}}{\boldsymbol{5}}$$

$$y+\frac{x}{10}+\frac{29}{5}$$

<u>H.W</u>

Find the equation of each of the tangent line and the normal line of the following function

1. $y=3x^2-2x$ $\quad at\ x=-1$
2. $y=\frac{x+2}{x}$ $\quad at\ x=2$
3. $y=\sin\left(\frac{\pi}{2}+x\right)$ $\quad at\ x=0$