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- NEYMAN-PERSON Theorem:-

$H_0: \theta = \theta_0 \quad , H_1: \theta = \theta_1$

if $\dfrac{L(H_1)}{L(H_0)} \geq k$ then we reject H_0

and the Best Critical Region (BCR) is:-

$C = \{x_1, \dots, x_n ; T(x) \geq k\}$

where k is fixed Positive number

$L(H_0)$ is the likelihood function under H_0.
$L(H_1)$ is the likelihood function under H_1.

Ex1) let $X \sim Exp(\theta)$, $n=2$, $H_0: \theta = 1$ vs $H_1: \theta = 2$
Find the BCR by N-P theorem.

SoL:-

$\because X \sim Exp(\theta) \Rightarrow f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$

N-P theorem is $\dfrac{L(H_1)}{L(H_0)}$

$n = 2$, $H_0: \theta = 1$, $H_1: \theta = 2$

$\rightarrow L(f(x,\theta)) = \left(\frac{1}{\theta}\right)^n \cdot e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$

$\rightarrow L(H_1) = \left(\frac{1}{2}\right)^2 \cdot e^{-\frac{1}{2}\sum_{i=1}^{2} x_i}$

$L(H_0) = \left(\frac{1}{1}\right)^2 e^{-\frac{1}{1}\sum_{i=1}^{2} x_i}$

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$$\Rightarrow \frac{L(H_1)}{L(H_0)} = \frac{\left(\frac{1}{2}\right)^2 \cdot e^{-\frac{1}{2}\sum_{i=1}^{2} x_i}}{\left(\frac{1}{1}\right)^2 e^{-\frac{1}{1}\sum_{i=1}^{2} x_i}} \geqslant k \quad \text{, we reject } H_0$$

$$\Rightarrow \frac{1}{4} e^{\left(-\frac{1}{2}+1\right)\sum_{i=1}^{2} x_i} \geqslant k \quad \text{, we reject } H_0$$

$$\Rightarrow \left[\frac{1}{4} e^{\left(\frac{1}{2}\right)\sum_{i=1}^{2} x_i} \geqslant k\right] \times 4$$

$$\rightarrow \left[e^{\frac{1}{2}\sum_{i=1}^{2} x_i} \geqslant 4k\right] \times \ln$$

$$\rightarrow \frac{1}{2}\sum_{i=1}^{2} x_i \geqslant \ln(4k) \quad \text{we reject } H_0$$

The Best Critical Region is

$$C = \{x_1, \dots, x_n ; T(x) \geqslant k\}$$

$$\Rightarrow \sum_{i=1}^{2} x_i \geqslant 2\ln(4k)$$

$$\therefore k^* = 2\ln(4k)$$

$$\Rightarrow \boxed{T(x) = \frac{1}{2}\sum_{i=1}^{2} x_i} \Rightarrow$$

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$$\therefore C = \left\{x_1, \dots, x_n ; \sum_{i=1}^{2} x_i \geqslant k^*\right\}$$

by dividing by $n = 2$ we get:-

$$\bar{x} = \frac{1}{2}\sum_{i=1}^{2} x_i \rightarrow \text{we get:- } \bar{x} \geqslant \ln(4k)$$

so if $k^* = \ln(4k)$ then the BCR will be

$$C = \{x_1, \dots, x_n ; \bar{x} \geqslant k^*\}$$

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Ex2) let $X \sim N(\theta, 1.69)$, $n=9$, $H_0: \theta=0$
Vs $H_1: \theta=1$, Find the BCR by N-P theorem.

SoL:-

$$\because X \sim N(\mu, \sigma^2) \qquad \mu=\theta , \ \sigma^2 = 1.69$$

$$X \sim N(\theta, 1.69)$$

$$N(\mu, \sigma^2) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$N(\theta, 1.69) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi \cdot (1.69)}} \cdot e^{-\frac{1}{2}\frac{(x-\theta)^2}{2(1.69)}}$$

$$L(f(x,\theta)) = \left(\frac{1}{2\pi \cdot (1.69)}\right)^{\frac{9}{2}} \cdot e^{\frac{-\frac{1}{2}\sum_{i=1}^{9}(x_i-\theta)^2}{2(1.69)}}$$

$H_1: \theta=1$
$H_0: \theta=0$

$$\frac{L(H_1)}{L(H_0)} = \frac{\cancel{\left(\frac{1}{2\pi(1.69)}\right)^{\frac{9}{2}}} \cdot e^{-\frac{1}{2}\frac{\sum_{i=1}^{9}(x_i-1)^2}{2(1.69)}}}{\cancel{\left(\frac{1}{2\pi(1.64)}\right)^{\frac{9}{2}}} \cdot e^{-\frac{\sum_{i=1}^{9}(x_i-0)^2}{2(1.69)}}} \geq K$$

we reject H_0.

$$\Rightarrow e^{\frac{-\cancel{\sum_{i=1}^{9} x_i^2} + 2\sum_{i=1}^{9} x_i - 1 + \cancel{\sum_{i=1}^{9} x_i^2}}{2(1.69)}} \geq K$$

$$\Rightarrow \frac{\sum_{i=1}^{9} x_i}{1.69} \geq \frac{1}{2(1.69)} + \ln(k)$$

We reject H_0

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Now if $K^* = 1.69\left(\frac{1}{2(1.69)} + \ln(K)\right)$

then the BCR will be :-

$$C = \{x_1, \dots, x_n ; \sum_{i=1}^{9} x_i \geq K^*\}$$

by dividing by $n = 9$ we get

$$\bar{x} \geq \frac{1.69}{9}\left(\frac{1}{2(1.69)} + \ln(K)\right)$$ we reject H_0

So if $K^* = \frac{1.69}{9}\left(\frac{1}{2(1.69)} + \ln(K)\right)$ then the BCR will be:-

$$C = \{x_1, \dots, x_n ; \bar{x} \geq K^*\}$$

H.W

1- let $X \sim P(\theta)$ is r.s of $n=5$
find BCR to test $H_0 : \theta = 2$ Vs $H_1 : \theta = 3$

2- For $X \sim Exp\left(\frac{1}{\theta}\right)$ r.s of size $n=2$, to test $H_0 ; \theta = 1$ Vs $H_1 ; \theta = 2$ Find the BCR

3- let $X \sim ber(\theta)$, to test $H_0 ; \theta = 0.2$ Vs $H_1 ; \theta = 0.3$ find the BCR.