# Methods of Point Estimation.

## 2- Moments Method.

* في هذه الطريقة تستخدم العزوم اللامركزية في عملية التقدير. حيث نقوم بوضع عدد من المعادلات مساوية لعدد المعلمات المطلوب تقديرها.

Definition:-

- let $X_1, X_2, \dots, X_n$ be a random sample from the probability distribution $f(x)$. where $f(x)$ can be a discrete probability mass function or a continuous probability density function. The Population moment (or distribution moment). is $E(X^k)$, $k = 1, 2, \dots$. The corresponding k-th sample moment is $(1/n) \sum_{i=1}^{n} X_i^k$, $k = 1, 2, \dots$

- let $X_1, X_2, \dots, X_n$ be a random sample from either a probability mass function or probability density function with $m$ unknown parameters, $\theta_1, \theta_2, \dots \theta_m$. The moment estimators. $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_m$ are found by equating the first $m$ population moments to the first $m$ sample moments and solving the resulting equations for the unknown parameters.

$$\mu_j = m_j(\theta_1, \theta_2, \dots, \theta_k) \quad , j = 1, 2, \dots, k$$

where $m_j = \frac{1}{n} \sum_{i=1}^{n} x_i^j$ be the $j^{th}$ sample moment.

and $\mu_j = E(x_i^j)$ be the $j^{th}$ population moment.

وعليه تكون المعادلات المتكونة من خلال العلاقات التالية.

$$1-\ \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x} = E[x]$$

$$2.\ \frac{1}{n}\sum_{i=1}^{n} x_i^2 = E[x^2]$$

$$3-\ \frac{1}{n}\sum_{i=1}^{n} x_i^3 = E[x^3]$$

$$\vdots$$

$$j-\ \frac{1}{n}\sum_{i=1}^{n} x_i^j = E[x^j]$$

Ex1) let $x_i \sim Beta(1,\theta)$; $i=1,2,--,n$. Estimate $\theta$ by moments method?

Solution:-

* هنا لنا معلمة واحدة مجهولة عليه ستكون بتكوين معادلة واحدة من معادلة رقم (1) اعلاه.

Since $x_i \sim Beta(1,\theta)$

$$\therefore E(x) = \frac{1}{1+\theta}$$

$$\rightarrow \text{from eq 1.} \Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i = E(x)$$

$$\bar{x} = \frac{1}{1+\hat{\theta}}$$

$$\Rightarrow \hat{\theta} = \frac{1}{\bar{x}} - 1 = \frac{1-\bar{x}}{\bar{x}}$$

; is the moment estimator for $\theta$.

**Ex2)** find the moments estimators for $\theta$ if

$$f(x,\theta) = \theta \cdot e^{-\theta x} \quad , x > 0.$$

**Solution)** we have one Parameter.

from eq 1. $\rightarrow \frac{1}{n} \cdot \sum_{i=1}^{n} x_i = E(x)$

$$\rightarrow E(x) = \int_0^\infty x \cdot f(x,\theta)\,dx = \int_0^\infty x \cdot \theta \cdot e^{-\theta x}\,dx = \frac{1}{\theta}$$

$$\rightarrow \bar{x} = \frac{1}{\hat{\theta}} \Rightarrow \hat{\theta} = \frac{1}{\bar{x}}$$ is the moment estimator for $\theta$.

**Ex 3)** let $x_i \sim N(\mu, \sigma^2)$; $i = 1, 2, \dots, n$. Estimate the Parameters $\mu$ and $\sigma^2$ by MO-method?

**Sol:-** we have two Parameters. $\mu$ and $\sigma^2$

$\rightarrow$ The first Eq. $\sum_{i=1}^{n} \frac{x_i}{n} = E(x)$

$\because x_i \sim N(\mu, \sigma^2) \quad \therefore E(x) = \mu$ , $V(x) = \sigma^2$

$$\rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i = \hat{\mu} \Rightarrow \boxed{\hat{\mu} = \bar{x}}$$

$\rightarrow$ The second Eq. $\rightarrow \frac{1}{n} \cdot \sum_{i=1}^{n} x_i^2 = E(x^2)$

Note that ; $V(x) = E(x^2) - [E(x)]^2 \Rightarrow E(x^2) = V(x) + [E(x$

$$\rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i^2 = V(x) + [E(x)]^2$$

$$\frac{1}{n} \sum_{i=1}^{n} x_i^2 = \hat{\sigma}^2 + \hat{\mu}^2 \Rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i^2 = \hat{\sigma}^2 + \bar{x}^2$$

$$\rightarrow \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \frac{n\bar{x}^2}{n} = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \delta^2$$